The 2021 Annual Report
National Center of Theoretical Sciences
Division of Mathematics

Contents

1 Overview

1.1 Report from the Director

The ongoing pandemic this year continues to disrupt many of the key operations of the National Center of Theoretical Sciences Mathematics Division (NCTS). One of the main consequences is that many of the previously planned international workshops and conferences had to be either cancelled, postponed, or held online in 2021. In fact, at the time of writing, the total number of visitors this year has dropped to less than 10% of the numbers in usual years. Nevertheless, the Center strives to make a difference in this difficult time and is happy to report some of 2021 highlights below.

1.1.1 Introduction

The NCTS Mathematics Division, hereinafter referred to as the Center, strives to be a research center of excellence in Asia. Its aim is to promote and enhance research in all important disciplines in mathematical sciences in Taiwan, and at the same time also to encourage new promising research directions that may have impact in the future development of mathematics.

Another important aim of the Center is for it to play a leading role in fostering talents in mathematical sciences and help training the next generation of scientific leaders in Taiwan. Although the Center is located on the campus of the National Taiwan University, the NCTS, as its name infers, is foremost a national center. It therefore needs to serve the mathematical community of all of Taiwan and provides a national platform for research collaboration. At the same time the Center is to be an international meeting place for stimulating discussions and exchange of new developments and ideas. To achieve this goal, the Center also endeavors to establish, cooperate and collaborate with other international research institutions. It has maintained close connections with many leading mathematical research institutions, e.g., MSRI in Berkeley, RIMS in Kyoto, KIAS in Seoul, PIMC in Vancouver and the Fields Institute in Toronto.

In this first year of Phase 5 of the NCTS, the Mathematics Division encounters several challenges. Many of them are related to the pandemic; the number of international visitors has dropped substantially compared to previous years. Because of this, virtually all international conferences and workshops had to be either postponed, cancelled, or moved online. Although online software and hardware have significantly improved over the last years, even the best ones are still no real replacement for direct personal interaction that is so important in mathematics. During the year we have been making a lot of efforts to improve the facilities and staff support for online activities. The Center has successfully moved many of its important programs online during this time, for example, its three educational programs, Taiwan Mathematical School (TMS), Undergraduate Research Program (URP), and Undergraduate Summer Research Program

(USRP). In collaborations with outstanding mathematicians from different countries, the NCTS has been hosting two very successful international online seminars. Our regular seminars also invite speakers from abroad to give online talks. This also opens new possibilities for the center: we are no longer restricted by limitations imposed by distance and time constraints of potential speakers.

The Center, as a national center, is to play a leadership role in research for the entire mathematical community in Taiwan. Therefore, in fulfilling its task, it is imperative that its operation involves and supports as many of the best mathematicians in Taiwan as possible. Starting from this new phase, a program committee is formed for each of the current 6 scientific programs. A program committee is the core for each program in charge of the planning and coordinating academic activities in the program. At the same time, these committees serve as consultants to the Executive Committee in important decisions. The two open proposals from last phase Harmonic Analysis and Data Analysis are now merged into Program D and F, respectively.

From July 29 to 30, the Center held the first NCTS Review and Strategic Development Meeting with its core members. Because of the pandemic the meeting was held online and was attended by all executive, academic, and program committee members and all the administrative staff members of the Center. The purpose of the meeting is for the members to share their experiences and gives the Center an opportunity to review, adjust and fine-tune its operation. This meeting will be an important annual event of the Center in the future and will further improve its operation.

1.1.2 Scientific Highlights

The research achievements in this very first year of this new phase are indeed very impressive. For example, some of the results obtained in Topical Program Number Theory and Representation Theory are breakthroughs in their fields and have received international recognition. They have been published in some of the very top mathematical journals such as Inventiones, Duke etc., that only accept articles that have exceptional impact on their fields.

We are also pleased that close affiliates of the Center who have been very actively involved in its operation continued to receive national recognition for their research achievements, e.g., in 2021 NCTS Center Scientist Chung-Jun Tsai has received the prestigious Ministry of the Science of Technology Outstanding Research Award, while two others, NCTS Center Scientist Hsin-Yuan Huang and NCTS Young Theoretical Scientist Wei-Fan Hu, have received the Wu-Ta You Award.

1.1.3 Academic Activities

To comply with the government imposed pandemic prevention measures, many previously planned scientific activities had to be either postponed, cancelled or moved

to an online platform. The number of such activities is therefore significantly lower compared to past years. Nevertheless, a total of 24 conferences and workshops were organized and hosted by the Center, in addition to 170 seminar talks this year at the time of writing. Special topic seminars are indispensable in research, and among the many scientific events this year we would like to highlight two online seminars: The NCTS International Geometric Measure Theory Seminar is a seminar organized jointly by NCTS Center Scientist Ulrich Menne and faculty members from University of Cambridge, New York University and Tokyo Institute of Technology, and it is held every two months. Among the invited speakers are some of the world foremost leading experts in the field, e.g, the speaker on September 22 this year was Fields Medalist Alessio Figalli. On the other hand, the Seminar of Algebraic Geometry in East Asia is a two-lecture seminar that takes place every two weeks. Among the organizers are former NCTS director Jungkai Chen and faculty members from University of Tokyo, Korean Institute of Advanced Study, University of Singapore, Chinese University of Hong Kong, Vietnam Academy of Sciences and Technology, Chinese Academy of Sciences and Kyungpook National University. This regular online seminar provides an important platform for interaction among algebraic geometers in East Asia during the pandemic.

In addition to those announced activities, some informal working or learning seminars are also organized to consolidate related backgrounds. To foster discussions and interactions within related fields, each program organizes once or two symposia (a day or two) every year. Some of activities particularly put special focus on those in their earlier career.

1.1.4 Postdoctoral Program

The NCTS postdoctoral program remains strong and continues to attract many outstanding applicants. The program is very international, with Taiwanese now forming a minority among all the postdoctoral fellows in the NCTS. There are 20 postdocs currently stationed at the Center from countries like the US, Japan, Germany, Italy, Korea, Turkey, Vietnam etc. Discussions and interactions among postdocs, even in different fields, are very active. Besides meeting with their mentors and participating in activities in their associated programs, NCTS postdocs are actively involved in the biweekly Taipei Postdoc Seminar which the Center jointly runs with the Institute of Mathematics Academia Sinica. The speakers are mainly selected from the large pool of postdoctoral fellows in the larger Taipei area and the lectures are given in an informal relaxed atmosphere.

Other activities such as Meeting with Directors, Spring Day and individual interviews are also arranged. The NCTS Spring Day is a one-day event, in which every postdoc is asked to give a talk on his/her recent works. On this occasion the annual NCTS Young Theoretical Scientist award ceremony is also held. The deputy director

conducts an individual interview with every postdoc every half a year to update on the progress and check on the needs of the postdocs.

1.1.5 NCTS Education

Because of the pandemic situation in May, the NCTS Undergraduate Summer Research Program (USRP) had to be conducted entirely online this summer. Nevertheless, the program has received many more applications than the Center is able to support, and consequently several excellent programs had to be turned down. A USRP program, as the name suggests, is a program in the summer in which 1-2 faculty members supervise a group of 2-4 undergraduate students to do original research in a topic agreed upon. An important factor in research is to be able to absorb new materials in a non-linear fashion, and the USRP and its counterpart URP (Undergraduate Research Program) are designed partially with this in mind to help undergraduate students reach this stage and introduce them to the process of solving original problems under the guidance of professional mathematicians. This year's USRP consists of 9 research programs and they cover a very wide spectrum of topics in mathematical sciences, from very pure traditional subjects such as number theory and geometric analysis to applied topics such as scientific computing, data analysis, and signal processing. Just like the USRP, the URP program starting in fall 2021 has received many more applications than the NCTS budget is able to support. Among the 17 applications 9 of them were selected by the Executive Committee of the NCTS. The topics of these programs again range from pure to applied mathematics.

In addition to these programs aimed at undergraduate students, the Center this year has organized 11 courses at the graduate level under its Taiwan Mathematics School (TMS) umbrella. TMS was created, partially also to address some of the challenges that science, as a discipline, faces in present day, e.g., the shrinking pool of students in Taiwan. There were also five NCTS short courses targeting graduate students.

To increase the impact of these programs and to inspire interested students and professors, the Center has been setting up individual homepages for TMS, USRP and URP under the category of NCTS Education. The TMS-Home page already contains a lot of materials in its current form. The USRP-Home page has just been created and we will continue fine-tuning it until its final form. We will come up with a similar one for URP soon.

The NCTS has a research assistant program that is open to college graduates who are interested in pursuing graduate studies abroad. The aim of the program is to better prepare highly motivated Taiwanese students for their graduate studies. The duration of a research assistant appointment is usually one year. Currently, there are 7 research assistants at the NCTS. Every NCTS research assistant is assigned a Center affiliated mentor who provides guidance and closely oversees the assistant's scientific progress. Selection is based on the candidate's potential and ability to pursue and complete a

doctoral program in mathematical sciences at a first rate research university. This program has been quite successful, as many of the program's almuni have been admitted to some of the very top graduate schools in the world.

We observe that the percentage of leading mathematicians in Taiwan who have received their Ph.D. degrees in Taiwan has significantly increased since the establishment of the NCTS in 1997. The NCTS has played an important role in their training and development and we believe that the NCTS Education programs will continue to have a significant impact on nurturing the next generation of scientists in Taiwan.

1.1.6 Future Plans

Looking ahead we hope that by next year the corona pandemic will be finally under control and the current travel restrictions imposed in most countries will be relaxed or maybe even removed. Hopefully many postponed conferences will be able to take place successfully. The complete list of planned activities can be found in each scientific program. We also have recently called for applications for 2022 Research Pairs and 2022 International Exchange Program. For visitor program, we will make special efforts on inviting renowned mathematicians to be our long term visitors with the hope that it will build up even closer and deep collaborations with the Taiwan mathematics community. Next year will be the 25th anniversary of the NCTS, and we would like to organize some celebrating academic activities.

From July 18, 2022 – July 29, 2022 the Center plans to host a joint summer school program with MSRI, Berkeley. The title of the summer school is *Recent Topics in Well Posedness* and it is organized by former Center director Prof. Junkai Chen along with Profs. Yoshikazu Giga (University of Tokyo), Maria Schonbek (University of California, Santa Cruz), and Tsuyoshi Yoneda (University of Tokyo). The purpose of the workshop is to introduce graduate students to fundamental results on the Navier-Stokes and the Euler equations, with special emphasis on the solvability of its initial value problem with rough initial data as well as the large time behavior of a solution. These topics have long research history. However, recent studies clarify the problems from a broad point of view, not only from analysis but also from detailed studies of orbit of the flow. This event was originally scheduled for this year, but was postponed due to the pandemic. Barring another unexpected outbreak we expect to have many attendants from abroad for this event next year.

Besides, the NCTS-MSRI summer school mentioned above, the MSRI in the summer of 2022 plans to organize 12 other summer schools, some of them in Berkeley while some others are held in Australia, Italy, and England. Usually, admission to MSRI summer schools is fiercely competitive and admitted students are highly motivated students from all over the world. The Center is eligible to nominate 2–4 students to attend these 2022 summer schools, and the MSRI will cover a substantial portion of travel support for these nominated students. Collaborations with MSRI and other in-

ternational research institutions are instrumental in training future scientists of Taiwan.

1.2 Summary of Data

In this section, we summarize some related data of the Center in 2021 until October, trying to give a glimpse on the structure and activities of the Center. Complete and detailed lists of the data can be found in the Appendix. We first begin with people in the Center. The appointment of a Center Scientist is for two years a term and at most for two terms. Young Theoretical Scientists can be appointed only once, and it is also for two years. The data of publication in Section 1.2.6 is collected until July of 2021.

1.2.1 Committees and Members

International Advisory Committee: Russel Caflisch (New York University), Hélène Esnault (Freie Universität Berlin), Shigefumi Mori (RIMS), Horng-Tzer Yau (Harvard University), Shing-Tung Yau (Harvard University), Robert Bryan (Duke University).

Executive Committee: Chiun-Chuan Chen (NTU), Jungkai Chen (NTU), Shun-Jen Cheng (AS), Ming-Lun Hsieh (AS), Yng-Ing Lee (NTU), Wen-Wei Lin (NYCU), Yuan-Chung Sheu (NYCU), Mao-Pei Tsui (NTU).

Academic Committee: Yi-Fan Yang (NTU), Ching-Jui Lai (NCKU), Nan-Kuo Ho (NTHU), Chun-Hsiung Hsia (NTU), Tsung-Ming Huang (NTNU), Je-Chiang Tsai (NTHU).

Topical Program Committees:

(A) *Number Theory and Representation Theory*: Yifan Yang (NTU, chair), Chieh-Yu Chang (NTHU), Chun-Ju Lai (AS), Fu-Tsun Wei (NTHU)

(B) *Algebraic Geometry*: Ching-Jui Lai (NCKU, chair), Chen-Yu Chi (NTU), Jeng-Daw Yu (NTU).

(C) *Differential Geometry and Geomtric Analysis*: Nan-Kuo Ho (NTHU, chair), River Chiang (NCKU), Chun-Chi Lin (NTNU), Chung-Jun Tsai (NTU).

(D) *Differential Equations and Stochastic Analysis*: Chun-Hsiung Hsia (NTU, chair), Jung-Chao Ban (NCCU), Chun-Yen Shen (NTU), Lung-Chi Chen (NCCU), Chi Hin Chan (NYCU).

(E) *Scientific Computing*: Tsung-Ming Huang (NTNU, chair), Wei-Fan Hu (NCU), Matthew M. Lin (NCKU), Ming-Cheng Shiue (NCTU), Suh-Yuh Yang (NCU).

(F) *Interdisciplinary Studies*: Je-Chiang Tsai (NTHU, chair), Te-Sheng Lin (NYCU), Hau-Tieng Wu (Duke), Yng-Ing Lee (NTU), Shun-Jen Cheng (AS).

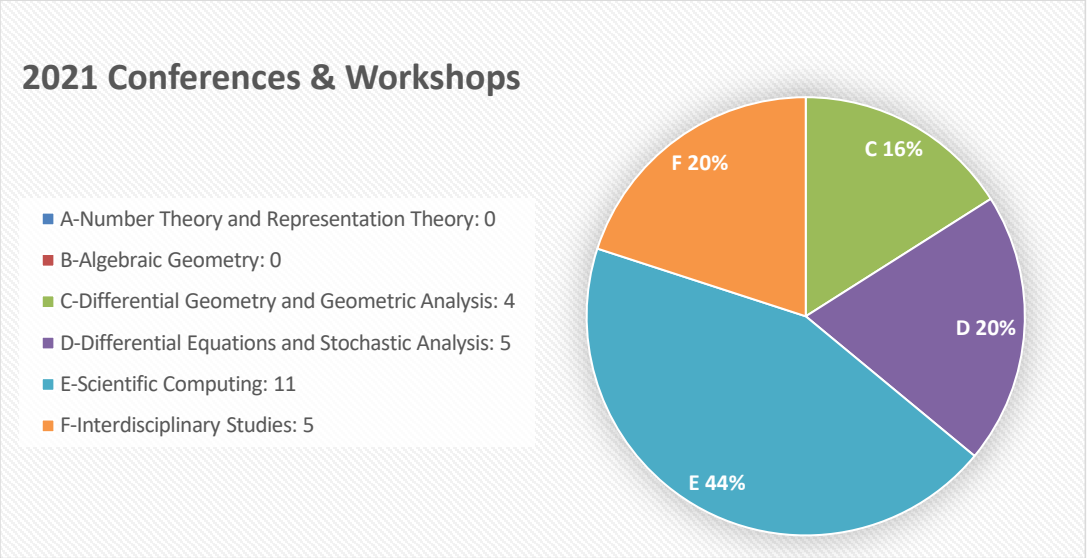
Center Scientists: Chung-Jun Tsai (NTU), Fu-Tsun Wei (NTHU), Chun-Yen Shen (NTU), Chi-Hin Chan (NYCU), Gi-Ren Liu (NCKU), Hao-Wei Huang (NTHU), Ulrich Menne (NTNU), Hsin-Yuan Huang (NYCU), Yung-Ning Peng (NCU).

Young Theoretical Scientists: Wei-Hsuan Yu (NCU), Wei-Fan Hu (NCU), Chih-Wei Chen (NSYSU), Chih-Whi Chen (NCU).

NCTS Scholars: Yujiro Kawamata (University of Tokyo), Gunther Uhlmann (University of Washington), Nikolaos Zygouras (University Warwick), Horng-Tzer Yau (Harvard University), Fan Chung Graham (UC San Diego), Paolo Cascini (Imperial College), Richard Schoen (UC Irvine), Kaoru Ono (Kyoto University), Keiji Oguiso (University of Tokyo).

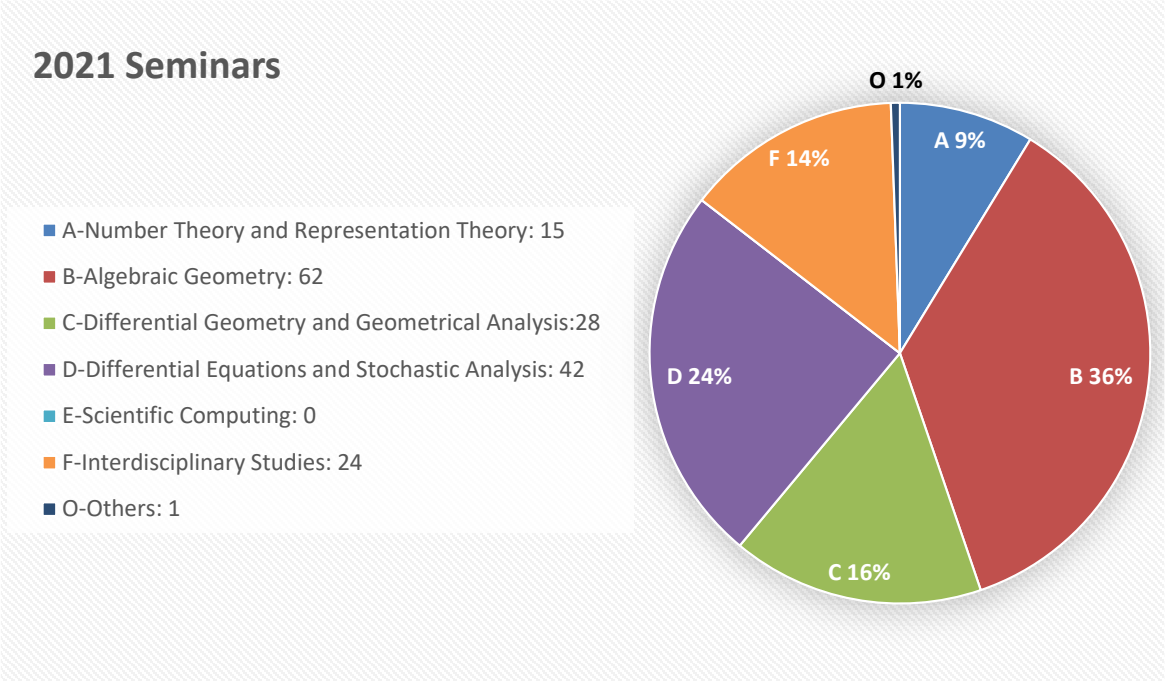
1.2.2 Workshops and Conferences

2021 Conferences & Workshops	
A-Number Theory and Representation Theory	0
B-Algebraic Geometry	0
C-Differential Geometry and Geometric Analysis	4
D-Differential Equations and Stochastic Analysis	5
E-Scientific Computing	11
F-Interdisciplinary Studies	5
	25



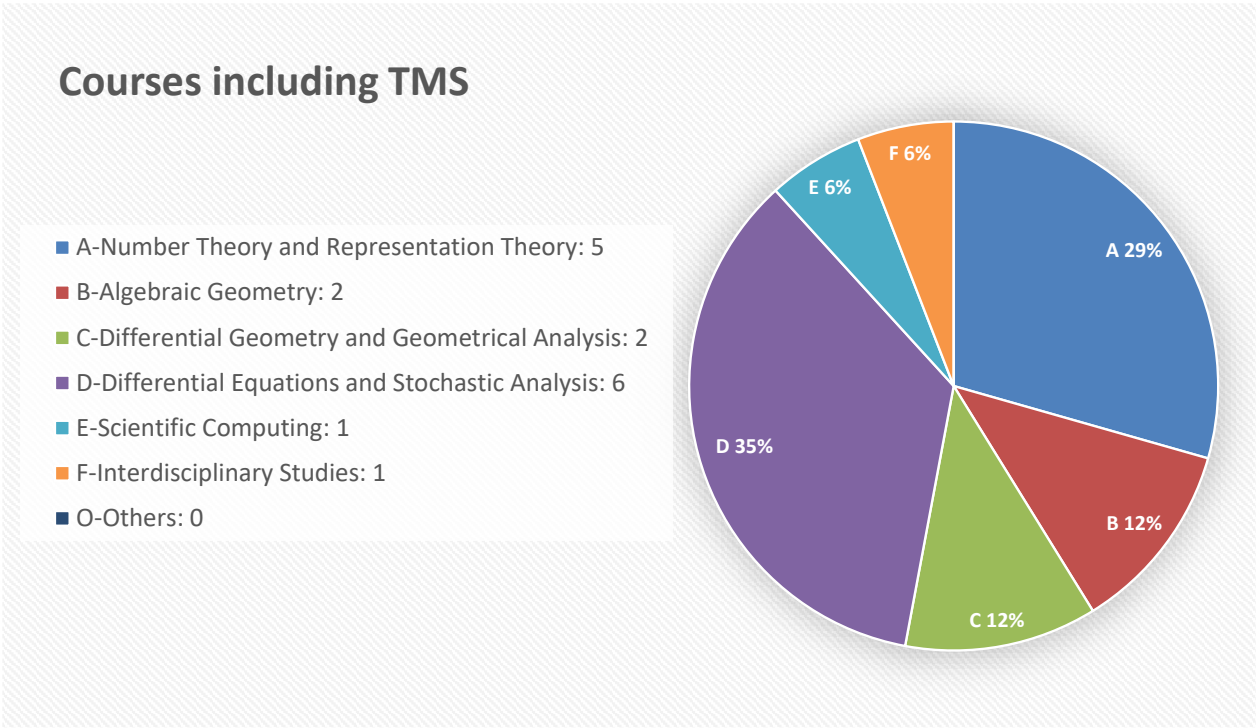
1.2.3 Seminars

2021 Seminars	
A-Number Theory and Representation Theory	15
B-Algebraic Geometry	62
C-Differential Geometry and Geometrical Analysis	28
D-Differential Equations and Stochastic Analysis	42
E-Scientific Computing	0
F-Interdisciplinary Studies	24
O-Others	1
	172

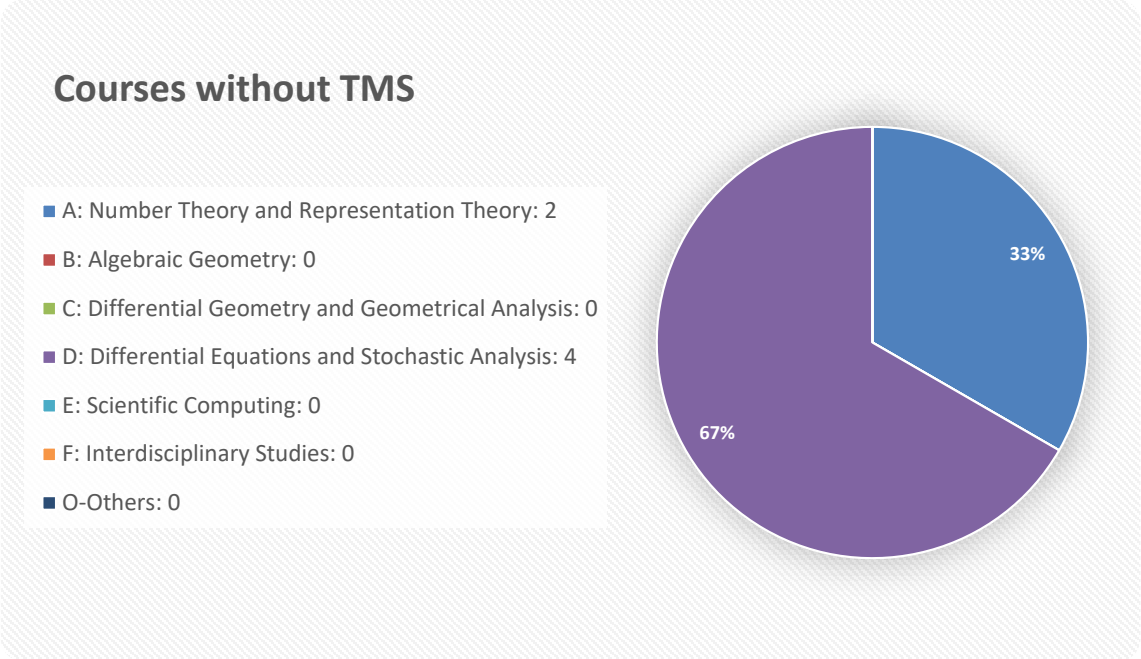


1.2.4 Courses

2021 Courses (including TMS)	
A-Number Theory and Representation Theory	5
B-Algebraic Geometry	2
C-Differential Geometry and Geometrical Analysis	2
D-Differential Equations and Stochastic Analysis	6
E-Scientific Computing	1
F-Interdisciplinary Studies	1
O-Others	0
	17

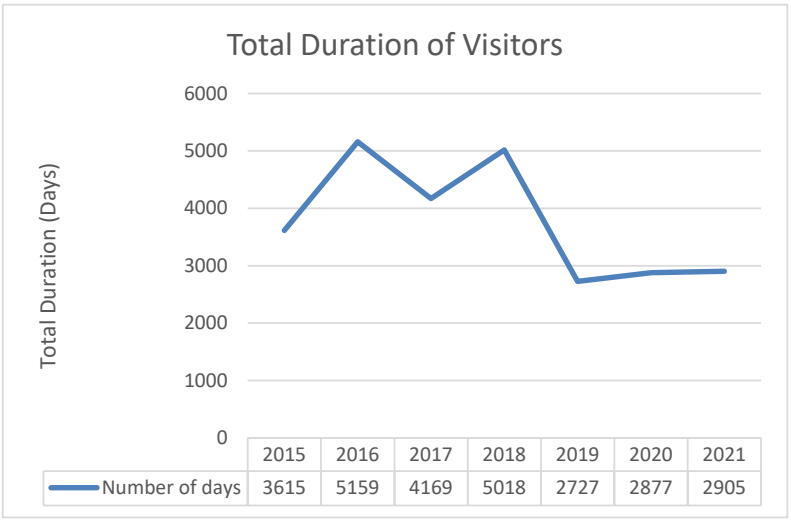


2021 Courses (without TMS)	
A: Number Theory and Representation Theory	2
B: Algebraic Geometry	0
C: Differential Geometry and Geometrical Analysis	0
D: Differential Equations and Stochastic Analysis	4
E: Scientific Computing	0
F: Interdisciplinary Studies	0
O-Others	0
	6



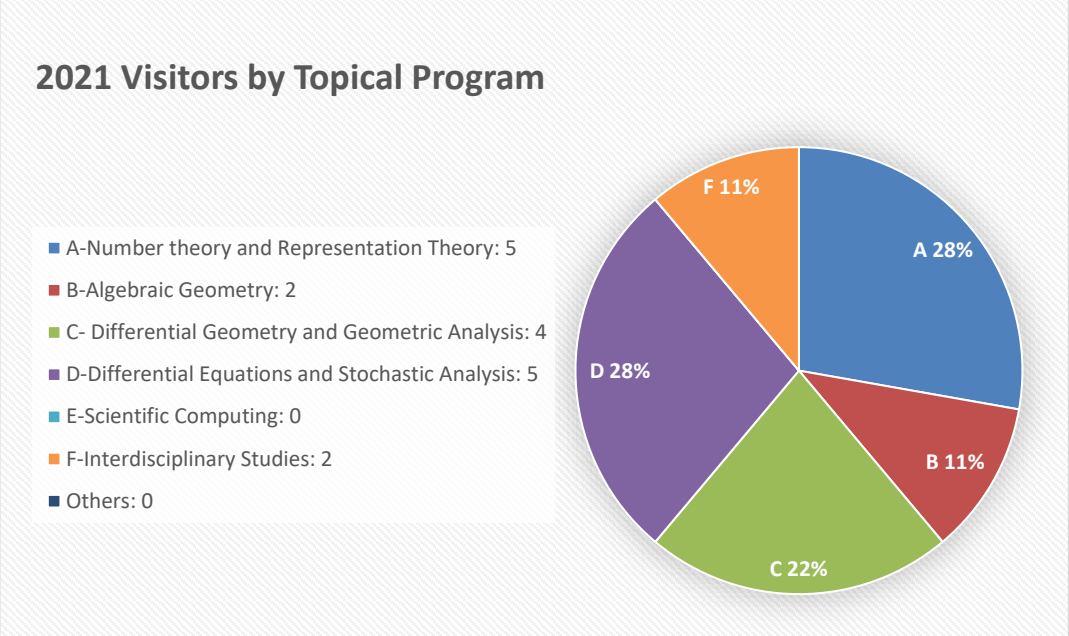
1.2.5 Visitors

Year	2015	2016	2017	2018	2019	2020	2021
Number of visitors	269	407	359	246	197	61	23
Number of days	3615	5159	4169	5018	2727	2877	2905



Visitors from Abroad

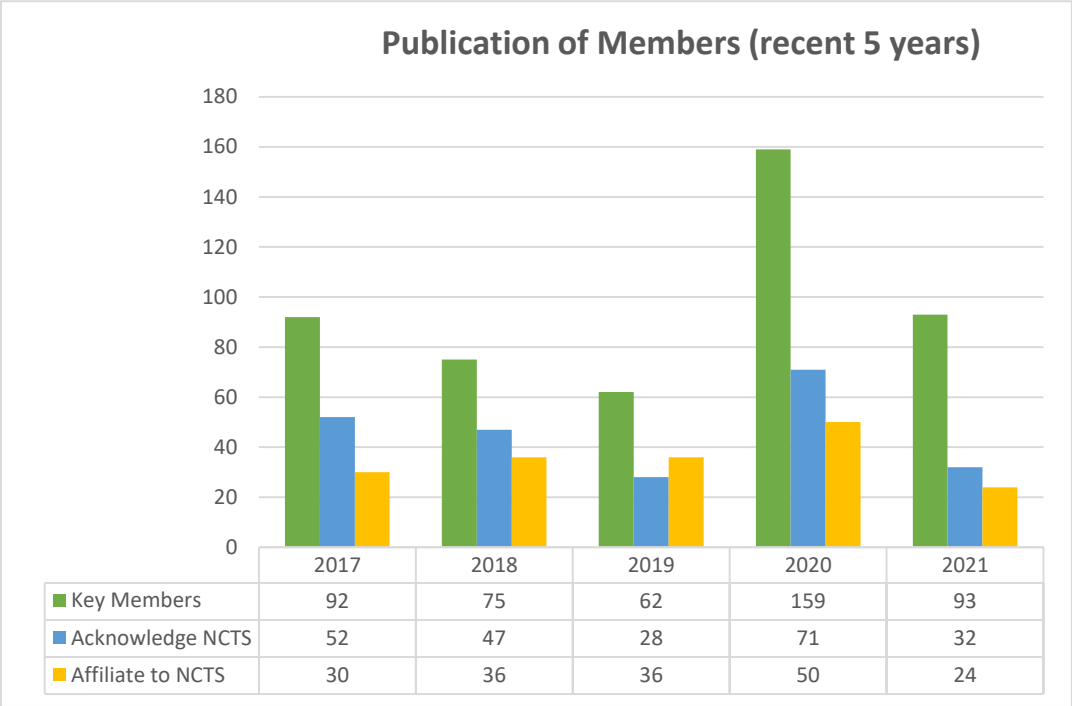
2021 Visitors		
A-Number theory and Representation Theory	5	28%
B-Algebraic Geometry	2	11%
C- Differential Geometry and Geometric Analysis	4	22%
D-Differential Equations and Stochastic Analysis	5	28%
E-Scientific Computing	0	0%
F-Interdisciplinary Studies	2	11%
Others	0	0%
	18	100%



2021 Visitors		
Country	Number	%
Japan	5	27.78%
USA	13	72.22%
	18	100.00%

1.2.6 Summary of publication data

The 2021 publications below are counted until July.



	2017	2018	2019	2020	2021
Key Members	92	75	62	159	93
Acknowledge NCTS	52	47	28	71	32
Affiliate to NCTS	30	36	36	50	24

2 Topical Programs

2.1 Number theory and Representation Theory

2.1.1 Core Members

1. Program Committee Members: Yifan Yang (NTU, chair), Chieh-Yu Chang (NTHU), Chun-Ju Lai (AS), Fu-Tsun Wei (NTHU).

2. Affiliated Faculty Members: Ming-Lun Hsieh (AS), Tzu-Yue Wang (AS), Chia-Fu Yu (AS),Ching Hung Lam (AS), Shun-Jen Cheng (AS), Nobuo Sato (NTU), Ming-Hsuan Kang (NYTU), Liang-Chung Hsia (NTNU), Chih-Whi Chen (NCU), Yung-Ning Peng (NCU), Wei-Hsuan Yu (NCU), Shih-Chang Huang (NCKU), Jia-Wei Guo (MOST independent researcher based at NTU).

3. Postdoctoral Fellows: Hang Fu (NTNU, then NTU), Oguz Gezmis (NCTS), Changningphaabi Namoijam (NTHU), Yasuhiro Terakado (AS, then NCTS), Kazuki Tokimoto (AS), Peng-Jie Wong (NCTS).

2.1.2 Program Overview

The purpose of the number theory and representation theory program at the NCTS is to assist domestic number theorists and algebraists to continue developing some active and promising research topics, and create opportunities for international cooperation as well as cultivating young researchers and students. Our research topics cover a wide spectrum of algebra and number theory from various perspectives. In the past year, our members focus on the following areas:

1. Iwasawa theory and p-adic methods in algebraic number theory and automorphic forms.

2. Special values over function fields and related topics.

3. Arithmetic and geometry of moduli spaces and explicit methods.

4. Arithmetic dynamical systems and Diophantine problems.

5. Representation theory of Lie superalgebras.

6. Vertex operator algebras (VOAs).

7. Hecke algebras and Cherednik algebras.

8. Geometric representation theory.

We run this program by having regular seminars on number theory and arithmetic geometry, organizing summer/winter schools, and hosting international workshops and conferences. The NCTS Number theory seminars are organized by C.-Y. Chang, and they are held in Taipei on Friday and in Hsinchu on Wednesday. Seminars on arithmetic geometry and representation theory are organized by C.-F. Yu and they were held in Taipei. These regular seminars provide a platform for research exchanges and discussions, and an opportunity for young scholars to broaden their horizons.

2.1.3 Research Highlights

In this section, we highlight some breakthrough works by our members in the past year.

1. Special values over function fields, *by C.-Y. Chang*.
   In recent years, I am devoted to the development of multiple zeta values (MZV's) over function fields in positive characteristic. In the joint work with Y. Mishiba [3], published in Inventiones 2021, we prove a function field analogue of Furusho's conjecture. More precisely, we show that there is a well-defined linear map from the ∞-adic MZV's to the v-adic MZV's with kernel containing the one-dimensional vector subspace spanned by the signal zeta value when the weight is "q"-even. Note that in the classical theory, Furusho's conjecture is still an open problem. In the preprint (https://arxiv.org/abs/2007.08264) with Y.-T. Chen and Y. Mishiba, we further generalize the above work in the sense that the map is indeed an algebra homomorphsim. Our ultimate goal is to prove the challenge that the kernel of the homomorphism is generated by the single zeta value at $q-1$ as is inspired by the spirit of Grothendick's period conjecture for real and p-adic multiple zeta values. The importance of the paper with Mishiba mentioned above is that we explicitly connect the special values MZV's from different non-archimedean worlds. And in fact, the transcendence tools that we used are parallel to the classical theory. That is, Yu's sub-t-module theorem that we used in the paper is the function field analogue of the celebrated Wüstholz's analytic subgroup theorem. However, the key ingredient is that we give logarithmic interpretation for MZVs, whose classical counterpart is not known yet. According to this logarithmic point of view, we shall be able to dig out more interesting results. For instance, in the joint paper with Green and Mishiba published by Math. Ann. [2], we can relate some Taylor coefficients of Anderson-Thakur series (analogue of motivic MZV's) to coordinates of the logarithm of a concrete t-module. These approaches will keep playing an important role in the study of special values over function fields in positive characteristic.

2. Arithmetic dynamical systems, *by L.-C. Hsia*.
In [1], we study the iterated Galois groups associated to unicritical polynomials defined over function fields of transcendence degree 1 over $\overline{\mathbb{Q}}$.

   Let K be the function field of a smooth irreducible curve defined over $\overline{\mathbb{Q}}$. Let $f(x) = x^d + c \in K[x]$ with $d = \deg f \geq 2$ and let $\beta \in \mathbb{P}^1(K)$. For $n \in \mathbb{N}$, the Galois group $G_n(\beta)$ of $f^n(x) - \beta$ can be shown to be embedded into $\mathrm{Aut}(T_n^d)$, the automorphism group of d-ary rooted tree T_n^d of level n, which is isomorphic to $[S_d]^n$, the nth iterated wreath product of the permutation group S_d on d letters. The inverse limit $G_\infty(\beta)$ of $G_n(\beta)$ is thus embedded into $\mathrm{Aut}(T_\infty^d)$, the automorphism group of an infinite d-ary rooted tree T_∞^d. It can be shown that $G_\infty(\beta)$ sits inside $[C_d]^\infty$, the iterated wreath product of the cyclic group C_d of order d. As an analogue of Serre's open image theorem, we show that for $d = p^r$, $r \geq 1$, a power of a prime p, and for $c \in K \backslash \overline{\mathbb{Q}}$, the image of $G_\infty(\beta)$ (in $\mathrm{Aut}(T_\infty^d)$ is of finite index in $[C_d]^\infty$ if and only if β is neither periodic nor in the orbits of the critical point under the action of f. Moreover, we show that if $f_i(x) = x^d + c_i$, $i = 1, 2$, are distinct polynomials, then the two towers of extensions associated to $\{f_i^{-n}(\beta)\}_n$, $i = 1, 2$, are disjoint over a finite extension of K.

3. p-adic automorphic representations, *by M.-L. Hsieh*.
In [7], we construct the three-variable p-adic triple product L-functions attached to Hida families of elliptic newforms and prove the explicit interpolation formulae at all critical specializations by establishing explicit Ichino's formulae for the trilinear period integrals of automorphic forms. Our formulae perfectly fit the conjectural shape of p-adic L-functions predicted by Coates and Perrin-Riou. As an application, we prove the factorization of certain unbalanced p-adic triple product L-functions into a product of anticyclotomic p-adic L-functions for modular forms. By this factorization, we obtain a construction of the square root of the anticyclotomic p-adic L-functions for elliptic curves in the definite case via the diagonal cycle Euler system á la Darmon and Rotger and obtain a Greenberg- Stevens style proof of anticyclotomic exceptional zero conjecture for elliptic curves due to Bertolini and Darmon.

4. Vertex operator algebras, *by C. H. Lam*.
Schellekens proved that the weight-one space V_1 of a strongly rational, holomorphic vertex operator algebra V of central charge 24 must be one of 71 Lie algebras. During the following three decades, in a combined effort by many authors, it was proved that each of these Lie algebras is realised by such a vertex operator algebra and that, except for $V_1 = \{0\}$, this vertex operator algebra is uniquely determined by V_1. Uniform proofs of these statements were given by

Möller-Scheithauer and Venkov.

In [14], we give a fundamentally different, simpler proof of Schellekens' list of 71 Lie algebras. We show that every strongly rational, holomorphic vertex operator algebra V of central charge 24 with $V_1 \neq \{0\}$ can be obtained by an orbifold construction from the Leech lattice vertex operator algebra V_A. This suffices to restrict the possible Lie algebras that can occur as weight-one space of V to the 71 of Schellekens.

Moreover, the fact that each strongly rational, holomorphic vertex operator algebra V of central charge 24 comes from the Leech lattice Λ can be used to classify these vertex operator algebras by studying properties of the Leech lattice. We demonstrate this for 43 of the 70 non-zero Lie algebras on Schellekens' list, omitting those cases that are too computationally expensive.

5. Hecke eigensystems of automorphic forms, *by Y. Terakado*.
In joint work with C.-F. Yu [13], I generalized Serre's mod p the Jacquet-Langlands correspondence to automorphic forms on Shimura varieties of Hodge type having good reduction at p. As an application, we gave an explicit and good upper bound of the number of the systems of Hecke eigenvalues of mod p automorphic forms on totally indefinite quaternionic PEL-Shimura varieties. This result suggests that automorphic forms in general settings also satisfy numerous congruence properties.

6. Automorphic representations over function fields, *by F.-T. Wei*.
In [15], I derive a generalized version of the Kronecker limit formula for GL_n in a conceptual way when n is arbitrary, and propose the "period" interpretations of various "Kronecker terms". This result enables me to prove a function field analogue of the Colmez conjecture for CM Drinfeld modules, and also applies to the arithmetic of special values of Rankin-Selberg and Godement-Jacquet L-functions associated to automorphic cuspidal representations of GL_n. This generalization gives rise to many subsequent problems, which are my current pursuit in my research topics.

7. Arithmetic of Shimura varieties, *by C.-F. Yu*.
In [8], jointly with Karemaker and Yobuko, we introduce the mass stratification on the supersingular locus S_g of the Siegel modular variety A_g for which the mass function is constant. An arithmetic significant is that this stratification controls the jumps of the automorphism groups and endomorphism rings of points

in S_g. For points in each mass stratum of S_3, they give an explicit mass formula and investigate possible automorphism groups. In particular, they show that the automorphism group of every point in the largest mass stratum has order two. This proves the Oort Conjecture on the automorphism groups of generic supersingular abelian threefolds.

In [12], jointly with Shen and Zhang, we construct the EKOR stratification in the special fiber of the Kisin-Pappas model. The EKOR stratification is the conjectural stratification described by work of He and Rapoport which interporates the EO strata when the level structure at p is hyperspecial and the KR strata in the Iwahori case. The main tools are the theory of G-zips and mixed characteristic local G-Shtukas. They also establish several geometric properties of EKOR strata including the smoothness, dimension formula and closure relation and use them to study Newton strata and central leaves.

8. Classification of supermodules over Lie superalgebras, *by C.-W. Chen*.
We establish a bijection between simple supermodules over an arbitrary type-I Lie superalgebra and simple supermodules over the even part of this Lie superalgebra. The construction of this bijection is very explicit and useful, namely, it is given by the so-called Kac induction functor. Consequently, this method reduces the classification problem for the supermodules over Lie superalgebras to the modules over Lie algebras. In addition, several criteria for the simplicity of Kac induced modules are also obtained in the paper [5].

9. Representation theory for Lie superalgebras, *by C.-J. Lai*.
The ultimate problem in representation theory of Lie algebras is to study the irreducible characters, whose answer is given by the Kazhdan-Lusztig (KL) theory in terms of the canonical bases for the Hecke algebras and for the quantum groups. The theory of canonical bases for quantum symmetric pairs have become a central topic in 21st century representation theory, as the 2020 Chevally Prize in Lie Theory went to Bao and Wang for their fundamental contributions to this theory and the application to the irreducible character problems for orthosymplectic Lie superalgebras. Part of my work is dedicated to study along this direction. This canonical basis theory was motivated mostly by Lusztig's work on quantum groups. Moreover, there is the so-called Beilinson-Lusztig-MacPherson (BLM) construction of (modified) quantum gl_n altogether with its canonical basis using the quantum Schur duality. In an earlier work, we have dealt with a generalization of BLM to affine type A. In [6], we established a BLM-type construction for affine type C. We utilize combinatorics of Young tableaux and affine Hecke algebras, which has the potential to be generalized to

the multiparameter case.

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2.1.4 Highlights of Events

Due to travel restrictions caused by the covid-19 pandemic, there were very few international visitors and all the international conferences and workshops were either cancelled or postponed in the past year.

Seminars

1. NCTS seminar on arithmetic geometry and representation theory.
   Organizer: C.-F. Yu.

2. NCTS seminar on number theory.
   Organizer: C.-Y. Chang.

3. NCTS seminar on combinatorics.
   Organizer: W.-H. Yu.

Lectures and Courses

1. Taiwan mathematics school: algebraic number theory I, fall, 2020 (C.-Y. Chang).

2. Taiwan mathematics school: algebraic combinatorics I, fall, 2020 (W.-H. Yu).

3. Taiwan mathematics school: mini-course on Springer fibers and quiver varieties, October 16–November 13, 2020 (Y.-N. Peng and C.-W. Chen).

4. NCTS winter course: six aspects of combinatorial mathematics, January 18–22, 2021 (W.-H. Yu and M. Fuchs).

5. Taiwan mathematics school: algebraic number theory II, spring, 2021 (C.-F. Yu).

6. Taiwan mathematics school: algebraic combinatorics II, spring, 2021 (W.-H. Yu).

7. NCTS summer course: polarized abelian varieties over finite fields, summer, 2021 (C.-F. Yu).

Workshops

1. One day combinatorics workshop, July 21, 2020.
   Organizer: W.-H. Yu.

2. Taiwan number theory symposium, September 9–11, 2020.
   Organizer: M.-L. Hsieh.

3. L-values of Iwasawa theory (online), November 11–12, 2020.
   Organizers: A. Burungale, M.-L. Hsieh, B. Palvannan, and S. Ramdorai.

2.1.5 Future Plans

Conferences and workshops:

In the upcoming year, we will organize three important conferences and several workshops. The first one is the 5th joint conference on number theory between Taiwan and Japan. Such a cooperation activity has been going on for several years, and the next edition will take place in Taiwan. The second one is the East Asia number theory conference. Due to travel restrictions, this edition of EANT will be held virtually. The third conference is a major international conference. IWASAWA is a series of conferences on the Iwasawa theory held every two years. It is one of the most important conferences on the subject. The list of confirmed speakers includes many renowned experts, such as J. Tilouine, M. Kakde, S. Dasgupta, F. Castella, G. Grossi, etc. Originally, it was scheduled to be held in Taiwan this year, but is postponed. (The dates are still to be decided.) In addition to the three conferences, we will organize several workshops, circumstance permitting, including the NCTS-Postech joint workshop on number theory, which has been held regularly for several years now, a workshop on arithmetic of function fields, and a workshop on multiple zeta values. Below is the list of activities mentioned above.

1. Japan-Taiwan joint conference on number theory.
   Organizers: M. Chiba, M.-L. Hsieh, and S. Kobayashi.

2. East Asia number theory conference.
   Organizers: S. Bae, L.-C. Hsia, Y. Taguchi, and F. Xu.

3. IWASAWA 2021.
   Scientific committee: D. Benois, H. Darmon, M.-L. Hsieh, M. Kurihara, O. Venjakob, and S. Zerbes.
   Local organizer: M.-L. Hsieh.

4. NCTS-Postech joint workshop on number theory.
   Organizers: J. Park and C.-F. Yu.

5. Workshop on arithmetics of function fields.
   Organizers: C.-Y. Chang and F.-T. Wei.

6. Workshop on multiple zeta values.
   Organizers: C.-Y. Chang and M. Kaneko.

7. Taiwan number theory symposium.
   Organizer: Y. Yang.

Seminars

1. NCTS seminar on arithmetic geometry and representation theory.
   Organizer: C.-F. Yu.

2. NCTS seminar on number theory.
   Organizers: C.-Y. Chang and Y. Yang.

3. NCTS seminar on combinatorics.
   Organizer: W.-H. Yu.

Courses and lectures:

In the next coming year, we plan to offer the following courses, ranging from beginning graduate courses in number theory to more specialized and advanced courses.

1. Analytic number theory. (P.-J. Wong and Y. Yang.)

2. Abelian varieties. (C.-F. Yu.)

3. Automorphic representations on $\mathrm{GL}(2)$. (M.-L. Hsieh.)

4. Hilbert modular surfaces. (C.-F. Yu.)

5. Iwasawa theory. (M.-L. Hsieh.)

6. Representation theory of finite-dimensional algebras. (C.-J. Lai and Z. Xiang.)

2.2 Algebraic Geometry

2.2.1 Core Members

1. Program Committee Members: Ching-Jui Lai (NCKU, chair), Chen-Yu Chi (NTU), Jeng-Daw Yu (NTU).

2. Affiliated Faculty Members: Jung-Kai Chen (NTU), Jheng-Jie Chen (NCU), Shin-Yao Jow (NTHU), Ting-Yu Lee (NTU), Chih-Chung Liu (NCKU), Jia-Ming Frank Liou (NCKU), Zhu Eugene Xia (NCKU), Yuan-Pin Lee (AS).

3. Postdoctoral Fellows: Chih-Wei Chang (NCTS), Bin Nguyen (NCTS), Iacopo Brivio (NCTS), David Wen (NCTS), Hsin-Ku Chen (NTU).

4. Students: Chi-Kang Chang (NTU, PhD), Shi-Xin Wang (NTU, undergraduate).

2.2.2 Program Overview

Algebraic geometry studies the geometry of algebraic and analytic objects and it has been a core area of the modern mathematics for more than a century. Today it remains a very active area in mathematics and related fields with many of its techniques, notions, and methods having important applications in number theory, differential geometry, physics, and even in statistics and data analysis. Thanks to the consistent and generous support of NCTS, the research group of algebraic geometry in Taiwan has maintained a good size and been active in various fields of study. Currently, the research directions of the algebraic geometry group at the NCTS have expanded significantly to the fields of birational geometry, generalized Hodge theory, linear systems and positivity, instantons and quiver varieties, analytical methods in algebraic and complex geometry, topology and geometry of moduli spaces, and nonabelian cohomology.

Through our past experience with NCTS, we continue to develop and deploy strategically various seminars, courses, and workshops. The main goals of the Topical Program of Algebraic Geometry currently is to broaden and solidify the research background of the researchers in Taiwan, to exploit and encourage domestic and international collaborations, and to cultivate the young generation to come in more efficient ways.

We would like to emphasize that Taiwan has a very strong working group in birational geometry. This has attracted many new postdocs to NCTS, including Iacopo Brivio from UCSD (former Ph.D. student of J. McKernan), David Wen from UCSB (former Ph.D. student of D. Morrison), Bin Nguyen from Universidade de Lisboa (former Ph.D. student of M.M. Lopes), and Chih-Wei Chang from NTHU (former Ph.D. student of S.-Y. Jow). In the coming academic year, we will also have Yen-An Chen from Univ. of Utah (former Ph.D. student of C. Hacon) to join us. Moreover, Prof. Jungkai Chen has established a research lab on algebraic geometry (with support from

MOST). Combining with the support of NCTS, this provides us a great chance to develop many more working activities and opportunities to interact with leading experts in the field.

2.2.3 Research Highlights

The selected research highlights of the program consist of the following parts: explicit birational geometry of threefold, Hodge theory, positivity of linear systems, $L2$ extension, and geography of special surfaces.

1. Explicit geometry of threefolds, by Jung-kai Chen, Jheng-Jie Chen, David Wen. Minimal model program (MMP) provides a unified way to understand the geometry of varieties, by creating a "minimal" representative in the birational class of a given variety. For projective surfaces, this leads to the Castelnuovo-Enriques classification in mid 19th century. However, the geometry gets much more complicated in higher dimensions and it is still an active research field. NCTS also has many researchers working in this direction and we highlight two recent achievement in this report.

   In dimension three, the early work of Mori, Reid, Kollár, et al has built a foundation on the understanding of singularities in MMP and birational maps. Since then, Jung-kai Chen at NCTS has been one of the leading experts on threefolds, especially for his work on minimal varieties of general type. For example, together with Meng Chen and Jiang Chen, they recently established an optimal Noether inequality for terminal threefolds of general type. The other problem is to extend the understanding of the puricanonical maps of a general type variety with small volume, which leads naturally to understand the geometry of special surfaces of ($K2 = 1$, $pg = 2$)-type. David Wen, a postdoc at NCTS, has worked in this direction and showed that the moduli space of $(1, 2)$-surface is irreducible with a unirational compactification. He also has also classified the automorphism group of these surfaces.

   The log canonical threshold is one of important and interesting invariants that measure singularities of pairs (X, B, S) that consist of algebraic varieties and boundary divisors on them. The ascending chain condition (ACC) conjecture on a-log canonical thresholds is one of major problems. In 2013, Hacon, McKernan, and Xu proved that the set of 0-log canonical thresholds satisfies the ACC in arbitrary dimension. This plays important role in the study of birational geometry. Mori fiber spaces are one of two outputs in the minimal model program. It is then natural and interesting to study the birational maps between two given Mori fiber spaces. Let $\phi : X \to X'$ be a birational map connecting two Mori fiber spaces $X \to S$ and $X' \to S'$. Sarkisov program tries to factor ϕ into

4 types of Sarkisov links which are expected to be more easily understood. In 1995, Corti showed Sarkisov program holds in dimension 3. The key measurement is the Sarkisov degree (μ, c, e) where $c = ct(X, H)$ denotes the canonical threshold of the pair (X, H). In fact, Corti's argument indicated almost immediately that birational Mori fiber spaces are connected by finitely many Sarkisov links if the set of threefold canonical thresholds satisfies the ACC. In the recent work of Jheng-Jie Chen, it is showed that the ACC for threefold canonical thresholds holds (=ACC conjecture on 1-log canonical thresholds in dimension 3 is true) in the case when $B = 0$ and S is an effective Weil divisor. That is, the set $T := \{ct(X, S)|dimX = 3, S \text{ is integral and effective}\}$ satisfies the ACC. Besides, the set T in the half interval $(1/2, 1)$ is described as follows: $T \cap (1/2, 1) = \{1/2 + 1/n|n \geq 3\} \cup \{4/5\}$.

2. Generalized Hodge theory and applications, by Jeng-Daw Yu. Hodge theory lies in the crossroads of frontier research in algebraic geometry, singularity theory, algebraic differential equations, theoretical physics and arithmetic. In the last decade we have developed the irregular Hodge theory, extending the classical Hodge theory. The new theory and its potential connections to other topics open many possible further research directions.

   With Fresán and Sabbah, we investigate the Hodge theoretic and exponential motivic aspects of power moments of Kloosterman sums. Using techniques involving irregular Hodge theory, étale cohomology and progress in the Langlands program, we determine the Hodge data and structures of the involved Galois representations. We prove that the associated L-functions are potentially automorphic. Particular significance of the work is to provide one of the first examples of applying the irregular Hodge theory and exponential motives to solve interesting and non-trivial problems for classical motives and in number theory. Furthermore, we investigate the periods of the motives associated with power moments of Kloosterman sums. Ingredients involve establishing theories of de Rham and singular (rapid decay) cohomology with support conditions for general connections and various dualities. Overall we obtain a fairly complete understanding of the arithmetic of moments of Kloosterman sums in the aspects of irregular Hodge theory and exponential motives.

3. Linear systems and positivity, by Shin-Yao Jow. Positivity properties of line bundles (e.g. ample, nef, big) are very important in algebraic geometry, c.f. Lazarsfeld's celebrated book. There have been some attempts to define and study positivity properties of vector bundles. For example, there is a fairly satisx factory theory of ample vector bundles due to Hartshorne. On the other hand, the theory of big vector bundles is far less developed, and many questions still remain. My current project involves studying the bigness property for toric vector

bundles. To get into more details, let X be a smooth complete variety, and let E be a vector bundle on X. Let $\pi : Y = P(E) \to X$ be the projective bundle of one-dimensional quotients of E , and let $L = O_Y(1)$ be the Serre line bundle on Y . We say that E is ample/nef/big on X if L is ample/nef/big on Y. If X is toric with dense torus T and character lattice M, a T-equivariant vector bundle E on X can be described by certain linear algebraic data called the Klyachko filtration. It is a collection of decreasing filtrations of the vector space $E =$ the fiber of E over the identity of T. Generalizing the polytope associated to a toric line bundle, Di Rocco, Jabbusch, and Smith use the Klyachko filtration of E to construct a polytope $P_e \subseteq M \otimes_{\mathbb{Z}} R$ for each nonzero vector $e \in E$. A goal of our project is to understand the relations between these polytopes and the Okounkov body of L, which should shed light on the bigness property of E.

4. Qualitative $L2$-extension from unions of strata of snc divisors, by Chen-Yu Chi. Let (E, h) be a holomorphic line bundle E on a complex manifold X equipped with a hermitian metric h, which is possibly singular. A main subject that has been studied for years is the possibility of extending a holomorphic section u of the restriction bundle $E|_S$ from a complex subspace S to X. This problem is largely motivated by the invariance of plurigenera and the abundance conjecture. Roughly speaking, qualitative $L2$-extension only cares about, provided u is $L2$, the existence of U but not the uniform norm estimate with respect to h in the so-called quantitative extension such as the various Ohsawa-Takegoshi type theorems. A general result of qualitative extension is obtained by Cao, Demailly, and Matsumura in 2015, but when the metric is singular, the center S in their result is not easy to describe in terms of algebraic geometry. In a recent work, Chen-Yu Chi obtains the following result: let X be a complex manifold admitting a projective morphism to a Stein space, $S = S_1 + \ldots + S_q$ a snc divisor in X, h_F a singular metric on a holomorphic line bundle F on X, and h_j some smooth metric on S_j. Under certain assumptions involving the curvatures of h_F and h_j and the singularities of h_F, a holomorphic section u on $(K_X + F + S)|_S$ which is $L2$ with respect to the metrics h and h_j's along every strata of S can be extend to a holomorphic extension U on X, which is locally $L2$ with respect to h and h_j's.

5. Canonical map of a surface of general type, by Ching-Jui Lai and Nguyen Bin. The problem of constructing examples of surfaces of general type with non-birational canonical map is motivated by the work of Beauville, which he undertook the study of the canonical map of surfaces of general type, bringing to light the great variety of possible behaviours of this map. In spite of the intriguing many examples which constructed by many authors in the last decade, several questions still remain open. The degree d of the canonical map of surfaces of

general type is at most 36. It is interesting to know which positive integers d occur as the degree of the canonical map of certain surfaces of general type. In recent times this question has been the subject of intense activity. While surfaces with $1 \leq d \leq 8$ are easy to construct, it is hard to find surfaces with $d \geq 9$. Only few surfaces with d greater than 8 have been known so far. The first surface with canonical map of degree higher than 8 was found by U. Persson in 1977 with $d = 16$. Later a surface with $d = 9$ was constructed by S. L. Tan in 1992. In the last decade, some surfaces with $d = 12, 24, 27, 32$ were constructed by Rito, Gleissner, Pignatelli. Surfaces with maximal $d = 36$ are constructed only recently by Ching-Jui Lai and Sai-Kee Yeung and Rito. We believe that there are many gaps to be filled. In the recent works, Nguyen Bin has been found some examples of surfaces with $d = 10, 14$ and with $d = 20$. All these work has extended our understanding to the geography of minimal surfaces of general type.

2.2.4 Highlights of Events

Due to the pandemic, we do not have any international visitors before August. Luckily, there are still many research activities conducted online.

1. *Seminar in Algebraic Geometry of East Asia.* Due to the global pandemic, most research activities are online. NCTS has joined and has been supporting one of these activities, the seminar in Algebraic Geometry of East Asia (AGEA). AGEA is a collaborative program conducted by institutions in China, Japan, Korea, Singapore, Taiwan, and Vietnam, which consists of regular research talks by algebraic geometers from all over the world. Through participating AGEA seminars, researchers in Taiwan can understand the research work in current trends and keep in contact with the global community. (The AGEA seminar is biweekly and started in August 2020. There have been more than 20 talks since.)

2. *NCTS Seminar on Algebraic Geometry.* This year, with our new group of organizing committee and many junior researchers, we decided to run a learning seminar on a selected advanced topic, with the aim of widening and solidifying research background in algebraic/complex geometry. We spent two weeks to solicit proposals and collected eight potential topics, including analytic method, valuation theory, arc spaces, mixed Hodge modules, deformation theory, rational curves, to tropical geometry. The two topics with the highest vote were then combined into a series of seminars titled "deformation theory and rational curves." Speakers and contents of each talk were chosen according to the proposals. In the end, we had 17 talks contributed by 10 researchers from February to July. We have covered most of the material in the books Deformation The-

ory by R. Hartshorne and Higher Dimensional Algebraic Geometry by O. Debarre. These topics are very useful for current research for some of our postdocs (working on liftability of varieties over positive characteristic) and also some junior faculties (working on Fano geometry). We consider this seminar to be a successful experience and plan to run it regularly.

2.2.5 Future Plans

In the future, we plan and continue to work on the following:

1. *Research exchange meetings and focused research groups.* We plan to organize a research exchange meeting, so that domestic researchers can discuss problems of their own interests and share points of view. We also encourage researchers to form focused research groups, to have regular meetings and share their working knowledge to the community. Based on these programs, we expect to build up a conventional format to encourage collaborations.

2. *Regular seminars, learning seminars, and lecture series.* We will continue to run learning seminars in the same form as we have done this year, to solidify and amplify fundamental background in algebraic/complex geometry of our researchers. We will also organize lecture series and workshops for exploring modern research.

3. *Training of students and Taiwan Math School.* We will continue to contribute to Taiwan Math School by offering courses, and in the long run establish a curriculum in algebraic geometry for training students in Taiwan.

4. *Workshops and Conferences.* Even in this difficult pandemic time, we will continue to encourage researchers to participate in international workshops and conferences if possible. We also encourage students and researchers to take short/long term trips for visits to major institutes in algebraic geometry.

5. *Undergraduate Research Summer Program.* We will consider problem-solving projects and also interdisciplinary research topics. Before URSP, some foundational background will be taught to students if necessary.

6. *Recruitment of world leading experts as NCTS distinguished scholars.* We will continue to recruit leading experts as NCTS distinguished scholars. With the help of these scholars, we will continue to have high quality workshops and better opportunities to exchange ideas for domestic researchers. This also helps to attract more researchers to visit NCTS and build international research collaborations.

2.3 Differential Geometry and Geometric Analysis

2.3.1 Core Members

1. Program Committee Members: Nan-Kuo Ho (NTHU, chair), River Chiang (NCKU), Chun-Chi Lin (NTNU), Chung-Jun Tsai (NTU).

2. Affiliated Faculty Members: Jih-Hsin Cheng (AS), Chin-Yu Hsiao (AS), Yng-Ing Lee (NTU), Mao-Pei Tsui (NTU), Shu-Cheng Chang (NTU), Siao-Hao Guo (NTU), Ulrich Menne (NTNU), Mei-Heng Yueh (NTNU), Rung-Tzung Huang (NCU), Ting-Jung Kuo (NTNU), Chiung-Jue Anna Sung (NTHU), Hung-Lin Chiu (NTHU), Siye Wu (NTHU), Dong-Ho Tsai (NTHU), Hsuan-Yi Liao (NTHU), Kuo-Wei Lee (NCUE), Eugene Zhu Xia (NCKU), Chih-Chung Liu (NCKU), Ryosuke Takahashi (NCKU), Ye-Kai Wang (NCKU), Chih-Wei Chen (NSYSU), Chung-I Ho (NKNU), Chin-Tung Wu (NPTU).

3. Postdoctoral Fellows: Ser-Wei Fu (NCTS), Andrea Galasso(NCTS), Jesse Madnick (NCTS), Yi-Sheng Wang (NCTS), Albert Wood (NCTS), Yuan Shyong Ooi (NTU), Jui-En Chang (NTU), Wei-Bo Su (AS), Kuang-Ru Wu (AS), Sheng-Fu Chiu (AS), Nicolau Sarquis Aiex (NTNU), Yang-Kai Lue (NCTU).

4. Students and Research Assistants: Yi-An Wu (NTU), Wei-Ting Kao (NTU), The Dung Tran (NTNU), Hsin-Chuang Chou (NTNU), Ting-Wei Tsui (NTNU), Chen-Kuan Lee (NTU), Jin Zhi Phoong (NTU), Minh-Wei Kuo (NTU), Ching-Chia Hsu (NTHU), Jiang Ting Chen (NTHU), Shuang Su (NCTS), Chin-Chia Chang (NCTS).

2.3.2 Program Overview

The aim of our program is twofold.

1. Create an engaging and thriving research environment for differential geometers in Taiwan: (a) To provide resources for interaction with international scholars for collaboration and developing future directions by holding special lecture series, international conferences, and hosting visitors. (b) To provide opportunity for interaction within the Taiwan community via holding seminars and symposiums.

2. Create a friendly and exciting learning environment for the young students. We wish to attract outstanding students to the field, and help them establish a solid foundation on the subjects as well as have some understanding of the important topics in the field by providing high quality courses and lecture series on current research topics.

The Program of Differential Geometry and Geometric Analysis covers a broad spectrum of themes, including a more analytic perspective as well as a more topological incarnation of modern geometry. Manifold learning and other applications are also introduced recently. Our current research directions cover:

a. Mathematical general relativity and minimal submanifolds

b. Geometric flows

c. Cauchy-Riemann Geometry and spectral geometry

d. Geometric measure theory and geometric variational problems

e. Geometric PDEs (metric structures)

f. Gauge theory, moduli space problems and mathematical physics

g. Symplectic and contact geometry

h. Discrete differential geometry and manifold learning theory

2.3.3 Research Highlights

a. Cauchy-Riemann geometry and spectral geometry by *Chin-Yu Hsiao (AS)*

The aim of the geometric quantization theory of Kostant and Souriau is to relate the classical observables (smooth functions) on a phase space (a symplectic manifold) to the quantum observables (bounded linear operators) on the quantum space (sections of a line bundle). The goal of the research at NCTS in this direction is to introduce Bergman/Szegő kernel technique to establish some fundamental results in geometric quantization theory. In [5], we showed that the G-invariant Szegő kernel for $(0, q)$ forms is a complex Fourier integral operator and we established quantization commutes with reduction theory for CR manifolds with transversal and CR S^1 action. We introduced for the first time the Fourier integral operator method to study geometric quantization theory. In particular, we gave a Szegő kernel proof for the classical Guillemin-Sternberg geometric quantization theorem on compact Kähler manifolds. Moreover, applying our results about G-invariant Szegő kernels asymptotics expansions to complex manifolds, we recover Weping Zhang and Xiaonan Ma's classical results about G-invariant Bergman kernels.

In quantum mechanics, the Bergman/Szegő kernel corresponds to density function of a particle in strong magnetic field on physical space (Kähler manifold), projected to the lowest Landau level and the study of Bergman/Szegő kernel asymptotics is closely related to the problem about fill up a domain with quantum states in quantum hall effect. The goal of the research at NCTS in this direction is to completely study

the asymptotic behavior of Bergman/Szegő kernel in some important geometric spaces (physical spaces). It is an old, open and difficult questions that if the Szegő and Bergman kernels admit an asymptotic expansions on the degenerate part of weakly pseudoconvex domains. The difficult of this kind of problem comes from the fact the microlocal analysis method does not work on the degenerate part of a CR manifold. In [4], we introduced "micolocal Harmonic oscillator" method, we constructed a pointwise Boutet de Monvel-Sjöstrand parametrix for the Szegő kernel of a weakly pseudoconvex three dimensional CR manifold of finite type and extends Fefferman's boundary asymptotics of the Bergman kernel to weakly pseudoconvex domains in $\mathbb{C}^2$. By this, we solved some old and open problems in several complex variables.

Let M be a compact Kähler manifold with Kähler form ω. The Kontsevich deformation quantization or star product for the algebra of $\mathcal{C}^\infty(M)$ with respect to ω is given by a power series of ν such that $*$ is an associative $\mathbb{C}[[\nu]]$-linear product. In 1994, Bordemann, Meinrenken and Schlichenmaier used Berezin-Toeplitz operator to construct star product on a projective manifold. In [1], we study the algebra of Toeplitz operators on a non-degenerate CR manifold X and we establish star product for some class of symbols on X. Applying our results to complex manifolds, we are able to construct star product on some class of non-Kähler manifolds.

b. Geometric measure theory, Geometric flows and minimal submanifolds

Ulrich Menne (NTNU)

For distributions, we build a theory of higher order pointwise differentiability comprising, for order zero, Łojasiewicz's notion of point value. Results in [6] include Borel regularity of differentials, higher order rectifiability of the associated jets, a Rademacher-Stepanov type differentiability theorem, and a Lusin type approximation. A substantial part of this development is new also for zeroth order. Moreover, we establish a Poincaré inequality involving the natural norms of negative order of differentiability. As a corollary, we characterise pointwise differentiability in terms of point values of distributional partial derivatives.

Chung-Jun Tsai (NTU)

In a Riemannian manifold, a minimal submanifold is the critical state of the induced volume functional. If the submanifold is one-dimensional, it coincides with the notion of geodesics. Studying such objects is a major approach to understand the relation between Riemannian geometric properties and topological properties of a manifold. In the case of geodesics, the existence, at least locally, is a direct consequence of the fundamental theorem of ODE. The behavior of the length functional near a geodesic is also well-understood; the Jacobi field equation can be analyzed by the ODE technique as well. When the dimension and codimension go higher,

the formulation becomes PDE. It drastically increases the difficulty. The mean curvature flow is the negative gradient flow of the volume functional, and is a natural proposal to construct minimal submanifolds. However, the evolution equation usually runs into singularity in finite time, and one does not expect a general parabolic PDE type result here.

In [7], we investigate the dynamical stability of a minimal submanifold under the mean curvature flow. We identify a natural curvature type condition on the minimal submanifolds, which is referred as strong stability. We prove that the condition implies a local rigidity result. The rigidity theorem is in the level of $\mathcal{C}^1$, and thus is stronger than the classical results from deformation consideration. The second implication of strong stability is that the minimal submanifold is indeed stable under the mean curvature flow, under $\mathcal{C}^1$ perturbation. G. Huisken showed that the singularity formation is governed by $\mathcal{C}^2$ quantities, and the $\mathcal{C}^0$ perturbation is in general too weak to work. The result generalizes our earlier work in the setting of special holonomy, and is sharp for the aforementioned reason.

Mao-Pei Tsui (NTU)

It is well-known that both uniqueness and stability for the solution to the minimal surface system do not hold in higher co-dimension in general. Most of the results for co-dimension one case don't work for higher co-dimensional case. There are not too many results in this directions In [8], we obtain the uniqueness of general codimension Dirichlet problem for minimal surface system in restricted classes. The condition is in terms of singular values and in particular covers the classical hypersurface case and earlier results in higher codimension. To prove the uniqueness result, a natural way is to consider the geodesic homotopy of two solutions. However, the singular values for linear combination of maps are not clear. We apply majorization techniques from convex optimization to overcome the difficulties.

c. Mathematical physics by *Siye Wu (NTHU)*

During 2020/21, my research in mathematical physics has been on the applications of differential geometry to quantum theory and relativity. In [12], I considered the fermionic counterpart of my work with A.Yoshioka on the deformation of the prequantum action in geometric quantisation. Like in the linear bosonic case, quantisation of fermionic systems also requires a choice of polarisation and there is a projectively flat connection on the space of such. The deformed prequantum action is shown to be compatible with this connection in geometric quantisation and the flat connection on the bundle of observables in deformation quantisation. The work demonstrates an interesting link between the two quantisation methods. In S2, I considered the quantisation of a family of bosonic and fermionic systems with varying phase space structures. This is different from the previous work on the quantisation of the same system with a family of polarisations. The novelty

is the observable geometric phases from the holonomy of the bundle of Hilbert spaces. The linear examples are the families of complex structures from a hyper-Kähler or paraquaternionic structure. In [10][11], in collaboration with L.Baulieu, NCTS (physics) visitor, et al, we proposed a second order Langevin equation in the stochastic quantisation of gravity. Classical gravity is described by general relativity. However, the failure of being bounded from below of its Euclidean action poses problems in quantum gravity, especially in stochastic quantisation, which extracts quantum effects from the equilibrium limit of the system in a random noise. The second order equation proposed in these two papers allows oscillatory behaviour in the stochastic time evolution and displays new features of quantum gravity.

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2.3.4 Highlights of Events

Due to the travel restriction caused by COVID19 pandemic, several activities involving overseas scholars have been cancelled or postponed. This pandemic posts a new challenge not only for us, but for every research institute around the globe. As traditional (in-person) international conferences appeared not to be feasible, activities had to go virtual. There are two types of virtual formats that have been implemented. The first one is a completely online format. For example, the international geometric measure theory seminar. The iGMT seminar uses a combination of various software (MS Teams, Miro, and Wonder) to allow social interactions during and after the seminar, apart from the lecture itself. The second one is a hybrid format. Thanks to relatively low infection cases in Taiwan (last year and earlier this year), we were able to hold in-person meetings for the Taiwan participants while international participants can join the conferences through online platforms. The workshop on geometric evolution equations and related fields in March 2021 is an example of this type. The activities in 2021 can be summarised as follows:

2.3.4.1 Workshop and conferences

a. Workshop on Geometric Evolution Equations and Related fields (March 8-9, 2021)

   Organizers: Shu-Cheng Chang (NTU), Martin Guest (Waseda), Yoshihiro Ohnita (Osaka), Mao-Pei Tsui (NTU).

   Geometric flows have been used to solve a variety of geometric, topological, analytical and physical problems. The main idea in geometric flows is to evolve some form of curvature by diffusive (heat type) equations such as the mean, inverse mean, Gauss, and Willmore flows of submanifolds, the Ricci, Kähler-Ricci, and Calabi flows of manifolds, the Yang-Mills and Hermitian-Einstein flows of connections and metrics on vector bundles, and the Yamabe and other conformal flows of metrics. Each of these flows seeks to evolve the corresponding geometric structures to canonical geometric structures. In this workshop, we will mainly focus on the following geometric evolution equations and related topics: Kähler Ricci flow, line bundle mean curvature flow, mean curvature flow, Sasaki-Ricci flow, inverse Monge-Ampere flow, harmonic map heat flows, network flow and elastic flow.

b. NCTS The 21st Taiwan Geometry Symposium at NTHU (May 1, 2021), and NCTS The 20th Taiwan Geometry Symposium at NCTS (November 7–8, 2020).

   Organizers: River Chiang (NCKU), Nan-Kuo Ho(NTHU), Chung-Jun Tsai (NTU) and Mao-Pei Tsui (NTU).

   This is a series of regular meetings aimed to foster discussions and interactions within the geometry community in Taiwan. It is held once every semester.

2.3.4.2 Seminars

One of the goals of our program is to create an engaging environment for the differential geometers in Taiwan. Thus, activities such as seminars are very important for exchanging ideas and stimulating interaction:

a. International seminar series: NCTS iGMT seminar

   Organizers: Ulrich Menne (NTNU), Guido De Philippis (Courant Institute, New York University, US), Yoshihiro Tonegawa (Tokyo Institute of Technology, Japan), and Neshan Wickramasekera (University of Cambridge, UK).

   This is an online seminar which is held every other month. It has been able to keep the quality of presentations high, because we keep the frequency of presentations low on purpose. The audience ranges from Japan and Taiwan in the East all the way to Europe and the East Coast of the US.

   The list of speakers is as follows:

   - Leon Simon (Stanford University), on 18th November 2020,
   - Juncheng Wei (University of British Colombia), on 20th January 2021,
   - Felix Schulze (University of Warwick), on 17th March 2021,
   - Tatiana Toro (University of Washington), on 19th May 2021,
   - David Bate (University of Warwick), on 21st July 2021, and
   - Alessio Figalli (ETH Zurich), on 22nd September 2021.

   On average, each presentation was attended by around 60 persons in total. The overall number of registered participants is steadily growing and has meanwhile surpassed 200 persons. Thus, this seminar is on a promising path to establish itself in the community.

b. Domestic Seminar Series: AS-NCTS seminar on Geometry and Several Complex Variables, NCTS Differential Geometry seminar, NCTS postdoc seminar for differential geometry group, and NCTS seminar on Geometry and Topology

2.3.4.3 Courses and lecture series

The aim of the lecture series is to provide an opportunity for researchers to learn about a current topic within a manageable time frame. in order to cultivate and foster outstanding students, NCTS has started the *NCTS Education* program.

a. Undergraduate student research program: Approaching modern geometric problems using geometric analysis, by Kuo-Wei Lee (NCUE) and Chih-Wei Chen (NSYSU), July 12– August 20, 2021

b. Taiwan Math School: Topics in Geometric measure theory II, by Ulrich Menne (NTNU), Spring 2021.

c. NCTS Mini Course on Manifold Learning at NCKU, by Chih-Wei Chen (NSYSU) and Mao-Pei Tsui (NTU), July 23 - 24, 2020.

2.3.5 Future Plans

The research interest of our members covers a vast array of topics. In the coming year, we would like to focus and promote some particular directions, hoping to further strengthen our group and establish excellent research teams in these fields.

They are:

a. Cauchy-Riemann geometry and spectral geometry

b. Geometric measure theory, geometric flow, minimal submanifolds, and general relativity

c. Symplectic geometry, gauge theory, and mathematical physics

d. Computational geometry and manifold learning theory

The reason for choosing these directions is based on the importance of the subjects as well as existing manpower in these fields.

As we stated earlier, our aim is to provide an engaging and thriving research environment for differential geometers in Taiwan and a friendly and exciting learning environment for the students. Thus, we plan our activities with these goals in mind, in the directions that we would like to promote and see fruitful growth.

In some perspective, travel restrictions may turn out to be a blessing in disguise. It is true that it affects the international activities greatly, however, as the virtual meeting software improve and people are getting more and more accustomed to using these software, it opens the opportunity for domestic seminars to have overseas speakers more often than pre-pandemic time.

Below is our plan for activities in the coming year, categorised according to directions:

a. For all

- The third Taiwan-Japan Joint Conference on Differential Geometry, November 1–3, 2021.

  Organizers: Shu-Cheng Chang (NTU), River Chiang (NCKU), Martin Guest (Waseda), Nan-Kuo Ho (NTHU), Yng-Ing Lee (NTU), Yoshihiro Ohnita (Osaka), Takashi Sakai (Tokyo Metropolitan University), and Mao-Pei Tsui (NTU).

  The purpose of the Taiwan-Japan Joint Conference on Differential Geometry is to develop collaboration, foster discussions and interactions between the differential geometry communities of Taiwan and Japan. The first one was held at Waseda University in 2016, the second one at NCTS in 2019, and the original plan for the third one was to be in Osaka; but due to the pandemic, it will be held virtually instead.
- One Taiwan Geometry Symposium each semester.

b. Cauchy-Riemann geometry and spectral geometry

- AS-NCTS seminar on Geometry and Several Complex Variables

c. Geometric measure theory, geometric flow, minimal submanifolds, and general relativity

- The 5th Taiwan International Conference on Differential Geometry.
  Organizers: Yng-Ing Lee (NTU), Chung-Jun Tsai (NTU), and Mao-Pei Tsui (NTU).

  This workshop highlights recent advances in global Riemannian and metric geometry, topology, and geometric analysis as well as identifies new directions of further research in these fields to set the stage for future advances.
- Seminar Series: NCTS iGMT seminar, NCTS Differential Geometry seminar, NCTS postdoc seminar for differential geometry group.

  For the coming year, we will keep focusing on geometry related to minimal submanifolds and special holonomy. We also plan to have more virtual talks by speakers within East Asia (Japan, Korea, and Hong Kong).
- NCTS lecture series on Ricci flow by Chih-Wei Chen (NSYSU) (originally scheduled for May 17, 24, and 31, postponed to Fall 2021)
- Taiwan Math School courses: Real Analysis by Ulrich Menne (NTNU)
- Short course on Ricci Soliton by Chih-Wei Chen (NSYSU), 2022.
- Visitors: Mu-Tao Wang (Columbia), Knut Smoczyk (Leibniz University Hannover, Germany), Pei-Ken Hung (University of Minnesota), Martin Man-chun Li (Chinese University of Hong Kong).

d. Symplectic geometry, gauge theory, and mathematical physics

- NCTS seminar on Geometry and Topology
- Taiwan Math School courses: Geometry and Quantum field theory by Siye Wu (NTHU)
- Lecture series on various topics in symplectic and contact geometry and topology. Organized by River Chiang (NCKU) and Kaoru Ono (RIMS, Kyoto).
- Lecture series on the non-abelian Hodge theory by Florent Schaffhauser (Universidad de Los Andes (Bogotá); University of Strasbourg).
- Visitors: Claus Hertling (Universität Mannheim, Germany), Martin Guest (Waseda), Yael Karshon (Toronto).

e. Computational geometry and manifold learning theory

- Manifold learning workshop, Spring 2022.

  Organizers: Chih-Wei Chen (NSYSU) and Mao-Pei Tsui (NTU)

  Manifold learning encompasses much of the disciplines of geometry, computation, and statistics, and has become an important research topic in data mining and statistical learning. The simplest description of manifold learning is that it is a class of algorithms for recovering a low-dimensional manifold embedded in a high-dimensional ambient space. This activity can be viewed as a follow-up to the mini course on manifold learning held on July 23-24, 2020, which provided an introduction to the geometric aspects of learning algorithms.
- Visitors: Albert Chern (UCSD), Guanglian Chen (San Jose State University).

2.4 Differential Equations

2.4.1 Core Members

1. Program Committee Members: Chun-Hsiung Hsia (NTU, chair), Jung-Chao Ban (NCCU), Chun-Yen Shen (NTU), Lung-Chi Chen (NCCU), Chi Hin Chan (NYCU).

2. Differential Equations

Chiun-Chuan Chen (NTU), I-Kun Chen (NTU), Jenn-Nan Wang (NTU), Hung-Wen Kuo (NCKU), Kung-Chien Wu, Ching-hsiao (Arthur) Cheng (NCU), Hsin-Yuan Huang (NYCU), Jin-Cheng Jiang (NTHU), Dong-Ho Tsai (NTHU), Jong-Shenq Guo (TKU), Jann-Long Chern (NTNU), Zhi-You Chen (NCUE), Jia-Yuan Dai (NCHU).

3. Dynamical System

Kuo-Chang Chen (NTHU), Chih-Hung Chang (NUK), Cheng-Hsiung Hsu (NCU), Yi-Chiuan Chen (Sinica).

4. Probability

Chii-Ruey Hwang (NCCU), Guan-Yu Chen (NYCU), Yuan-Chung Sheu (NYCU), Hao-Wei Huang (NSYSU), Shang-Yuan Shiu (NCU), Gi-Ren Liu (NCKU).

5. Harmonic Analysis

Chin-Cheng Lin (NCU), Daniel Spector (NTNU).

2.4.2 Program Overview

The study of differential equations is one of the most traditional fields of mathematics in Taiwan. The NCTS Topical Program Differential Equations and Stochastic Analysis also aims to incorporate interdisciplinary research as one of its priorities, and its four main directions are: partial differential equations, dynamical systems, stochastic analysis and harmonic analysis. Partial differential equations are one of the most widely adopted mathematical tools in modern sciences. Members of this NCTS Topical Program group are interested in both the developments of the mathematical theory and scientific applications. This includes, for example, synchronization problem, kinetic theory, elliptic partial differential equations, fluid dynamics, inverse problem and reaction-diffusion equations. Interests of the research groups in dynamical systems include bifurcation theory, chaotic systems, ergodic theory, fractal geometry, arithmetic dynamical systems, and complex dynamical systems. The research interests of the harmonic analysis group include two weights problems for singular integrals, additive combinatorics and their connections to geometric measure theory. Probability theory plays an important role in many other areas of mathematics, such as partial differential equations, analysis, and combinatorics. It provides the theoretical basis for statistics. In stochastic analysis at the NCTS, focus is placed on statistical mechanics, mathematical biology, finance, theoretical computer science, branching processes and SPDE.

2.4.3 Research Highlights

a. Differential Equations

(1) Chiun-Chuan Chen and his collaborators investigated the existence and stability of non-monotone travelling wave solutions for the diffusive Lotka-Volterra system of three competing species. They considered the problem: if coexistence occurs in the long run when a third species w invades an ecosystem consisting

of two species u and v on $\mathbb{R}$, where u, v and w compete with one another. Under the assumption that the influence of w on u and v is small and other suitable conditions, they showed that the three species can coexist as a non-monotone travelling wave. Such type of non-monotone waves plays an important role in the study of three-species phenomena. However, only few results are known for the existence of such waves in the literature. Their approach, based on the method of sub- and super-solutions and bifurcation theory, provides a new approach to construct non-monotone waves of this type. Moreover, they proved that the waves constructed are stable. This is the first rigorous result of stability for such waves.

(2) In collaboration with Magdalena Czubak, Chi Hin Chan has successfully investigated the so called restriction argument of the standard Laplacian acting on a vector field upon an embedded hypersurface in the Euclidean space. This study lies in the fact that, in the important case of the ellipsoid, we start with a 3D- vector field tangential to the ellipsoid and convert it to a 1-form, and let the Hodge Laplacian acts on it. Then, they restrict the resulting object upon the ellipsoid. The key contribution is that they succeed in phasing or converting every single term as involved in this restriction of the Hodge Laplacian of the 1-form associated to a tangential vector field, in a purely geometric invariant way, which is totally independent of the coordinate representation of the ellipsoid.

(3) Jong-Shenq Guo and his collaborators investigate the traveling wave solutions of a three-species system involving a single predator and a pair of strong-weak competing preys. It shows that here exist traveling waves where the strong prey invades the environment and either replaces its weak counterpart, or more surprisingly the three species eventually co-exist.

(4) Joint with his collaborator Phillipo Lappicy, Jia-Yuan Dai adopted an innovative shooting method to derive dynamical information on spiral waves, such as hyperbolicity, unstable dimension, and characterization of the global attractor. Those obtained spiral waves serve as ideal reference solutions for secondary bifurcations to more intricate spatio-temporal patterns, for instance, meandering and even drifting spirals.

(5) Chun-Hsiung Hsia and his collaborators studied a singular perturbation problem from the artificial compressible system to the incompressible system for a doubly diffusive convection when a Hopf bifurcation from the motionless state occurs in the incompressible system. It is proved that the Hopf bifurcation also occurs in the artificial compressible system for small singular perturbation parameter, called the artificial Mach number. The time periodic solution branch of the artificial compressible system is shown to converge to the corresponding bifurcating branch of the incompressible system in the singular limit of vanishing artificial Mach number.

(6) Kung-Chien Wu and his collaborators studied stationary boundary-value problem for the Boltzmann equation in a half space for a binary mixture of gases when the indata on the boundary are given for the both species. Under the assumption that one of the species is dominant and close to equilibrium but the density of the other is small, the problem is decomposed into two half-space problems: the so-called Milne problem for the linearized Boltzmann equation with a source term for the dominant species and that for the linear Boltzmann equation for the low-density species. The existence and uniqueness of the solutions to the two problems are proved, and their accurate asymptotic behavior in the far field is obtained. In particular, the precise rate of approach of the solution to the state at infinity is expressed in terms of the decay rate in the molecular velocity of the boundary data for both species.

b. Dynamical System

(1) Jung-Chao Ban, Chih-Hung Chang and their collaborators investigated the behavior of shift spaces on a class of semigroups and groups. They revealed the mixing properties related to the existence of Devaney chaos on these algebraic systems. They also studied the topological entropy and entropy dimension of shifts of finite type on semigroups in addition to characterizing their topological dynamics. Related papers are published in J. Algebraic Combinatorics, J. Differential Equations, J. Mathematical Physics, J. Statistical Physics, Nonlinearity, and Trans. Amer. Math. Soc.

(2) Cheng-Hsiung Hsu and his collaborators examined the existence and stability of the traveling wave solutions of reaction-diffusion equation, include the illustration of periodic wavefronts. Recently, they attained some theorems about the entire solutions for delayed nonlocal dispersal system with monostable nonlinearities. Related papers are published in J. Dynamics Differential Equations, J. Differential Equations, and Nonlinearity.

(3) Kuo-Chang Chen and his collaborators considered the possible shapes of central configurations. They proved the existence of convex but not strictly convex central configurations for the planar five-body and spatial seven-body problems. They also characterized strictly convex planar five-body central configurations in terms of mutual distances and the variational principle on heteroclinic orbits of n-body problems. Related papers are published in Cal. Var. Partial Differential Equations and Trans. Amer. Math. Soc.

c. Probability

(1) Lung-Chi Chen and his collaborator consider the long-range models on $\mathbb{Z}^d$ of random walk, self-avoiding walk, percolation and the Ising model, whose

translation-invariant 1-step distribution/coupling coefficient decays as $|x|^{-d-\alpha}$ for some $\alpha > 0$. In our previous work in 2015, we have shown in a unified fashion for all $\alpha \neq 2$ that, assuming a bound on the "derivative" of the n-step distribution (the compound-zeta distribution satisfies this assumed bound), the critical two-point function $G_{p_c}(x)$ decays as $|x|^{\alpha \wedge 2-d}$ above the upper-critical dimension $d_c \equiv (\alpha \wedge 2)m$, where $m = 2$ for self-avoiding walk and the Ising model and $m = 3$ for percolation. In this paper, we show in a much simpler way, without assuming a bound on the derivative of the n-step distribution, that $G_{p_c}(x)$ for the marginal case $\alpha = 2$ decays as $|x|^{2-d}/\log|x|$ whenever $d \geq d_c$ (with a large spread-out parameter L).

(2) There has been intensive research on free Lévy process. Hao-Wei Huang and his collaborator contributed to this research topic by proving strong regularity results for free additive and multiplicative Lévy processes. Motivated by the use of regularizing properties of free diffusions and non-commutative entropies, they studied the so-called property (H) introduced by Belinschi, Benaych-Georges and Guionnet. Necessary and sufficient conditions for the free additive and multiplicative Lévy processes having this property are offered. The problem about the number of components in the support of Lévy processes was also considered. Based on elementary integration theory, they constructed concrete examples with or without the property (H). Huang's another research line was regarding the limit theorem in bi-free probability theory. Since the invention by Voiculescu in 2013, there have been rapid developments made combinatorically and analytically in this new theory. Huang and his collaborator continued the research on the bi-free limit theorem from the perspective of analytic machinery, especially on linking the classical and bi-free limit theorems of infinitesimal selfadjoint and unitary faces. All results discovered by them essentially relied on the tool of bi-free harmonic analysis. As expected, it was shown that the classical limit theory of infinitely divisible laws, due to Lévy and Khintchine, and the bi-free limit theory of infinitely divisible laws parallel perfectly.

(3) Gi-Ren Liu and his collaborator analyzed the scaling limit of random processes arising from the second-order scattering transform coefficients of Gaussian processes. They proved that the rescaled random processes converge to the absolute value of Gaussian processes for commonly used wavelets, including the Morlet wavelet, the Mexican hat wavelet, and the Daubechies wavelet. For wavelets, whose Fourier transform tends to zero slowly in the neighborhood of the origin, e.g., the Morse wavelets, we proved that the rescaled random processes converge to the absolute value of non-Gaussian processes under a non-CLT type normalization. In brief, the current study focused on finding the limiting law of the rescaled outputs of the scattering transform. It gives an opportunity for us to understand the scattering transform from a probabilistic

viewpoint.

(4) Kyung-Youn Kim and her collaborator considers non-isotropic Markov process and estimates the upper and lower bounds for the transition density of the processes. They consider the d-dimensional Lévy process $L = (L^1, \ldots, L^d)$ where L^i are independent processes with jumping kernels $J_\phi(x^i, y^i) = \phi(|x^i - y^i|)^{-1}|x^i - y^i|^{-1}$. Here $\phi(r)$ is an increasing function with weakly scaling condition. Using the jumping kernel J comparable to $\sum_{i=1}^d J_\phi$, they prove the existence of the Markov process M corresponding to J, and obtain the sharp two-sided transition density's bound for M. By Chen and Bass, the non-isotropic process with the jumping kernel $|x^i - y^i|^{-d-\alpha}$ is studied at the first time, and then, there is a research on the regularity of transition density. Very recently, there is a result on the sharp two-sided transition density estimates by Kassmann, Kim and Kumagai is a generalized result covering a very large class of Markov processes using the function ϕ.

d. Harmonic Analysis

(1) Chun-Yen Shen and his collaborators, in a series of two weights projects, they proved a very general two weight boundedness criteria for any fractional singular integrals, including the Cauchy singular integral in the complex plane, that when one of the measures is supported on a curve and the other measure is arbitrary in terms of $T1$ conditions. In addition, the ultimate Tb criterion has also been proved to be held for Hilbert transform.

(2) Daniel Spector and his collaborators, in an on-going series of papers, have established new Lebesgue and Lorentz space estimates for solutions of partial differential equations (PDE) with L^1 data. The type of PDE considered are Div-Curl, or more generally, Hodge systems, and arise in the study of electricity and magnetism. The analogous estimates for L^p data, $p > 1$, are classical, and can be proved using soft analysis arguments. However, the assumption the data is in L^1 is the natural physical one, and requires different techniques.

2.4.4 Highlights of Events

We briefly summarize some of the most interesting and important activities held in 2021 as follows.

(1) 2021 NCTS Young Dynamics Day (2021/2/19) at TKU. Organizers: Jung-Chao Ban (NCCU), Chih-Hung Chang (NUK), Kuo-Chang Chen (NTHU), Cheng-Hsiung Hsu (NCU), Ting-Hui Yang (TKU).

(2) 2021 Dong Hwa Workshop in Probability and related fields (2021/2/26–27). Organizer: Chia-Li Wang (NDHU).

(3) The 11th Japan-Taiwan Joint Workshop for Young Scholars in Applied Mathematics (2021/03/08–2021/03/09, Online). Organizers: Chueh-Hsin Chang (THU), Chiun-Chuan Chen (NTU), Yan-Yu Chen (NTU), Jann-Long Chern (NTNU), Yung-Fu Fang (NCKU), Chang-Hong Wu (NYCU).

(4) Third NCTS PDE Symposium (2021-03-26) at NUK. Organizers: Chun-Hsiung Hsia (NTU), Ying-Chieh Lin (NUK), Tsung-Fang Wu (NUK), Kung-Chien Wu (NCKU).

(5) Nonlinear Phenomena in Evolutionary Partial Differential Equations (2020/9/22–2021/7/31) at NTU. Organizers: Shih-Hsin Chen (NTU), Jia-Yuan Dai (NCTS), Shun-Chieh Wang (NTU).

(6) NCTS-South Taiwan Seminar on Dynamical Systems (2020/9/24–2021/1/14) at NUK. Organizers: Jung-Chao Ban (NCCU), Chih-Hung Chang (NUK).

(7) NCTS Nonlinear PDE seminar (2021/3/3–2021/5/1) at NTU. Organizers: I-Kun Chen (NTU), Chun-Hsiung Hsia (NTU).

2.4.5 Future Plans

(1) NCTS Probability summer school. Organizer: Lung-Chi Chen (NCCU).

(2) NCTS PDE seminar, Fall 2021–Spring 2022. Organizers: I-Kun Chen (NTU), Chun-Hsiung Hsia (NTU).

(3) Nonlinear Phenomena in Evolutionary Partial Differential Equations at NTU (2021/9/22 – 2022/7/31). Organizers: Shih-Hsin Chen (NCTS, NTU), Shun-Chieh Wang (NTU).

(4) NCTS Webinar Nonlinear Evolutionary Dynamics (Fall 2021–Spring 2022). Organizers: Jia-Yuan Dai (NCHU), Chih-Chiang Huang (CCU), Chang-Hong Wu (NYCU).

(5) International online conference for kinetic theory, Spring 2022. Organizer: Kung-Chien Wu (NCKU).

(6) NCTS Differential Equations and Stochastic Analysis Young Scholar Workshop, Fall 2021 or Spring 2022. Organizers: Jung-Chao Ban (NCCU), Lung-Chi Chen (NCCU), Chun-Hsiung Hsia (NTU), Kung-Chien Wu (NCKU), Chun-Yen Shen (NTU).

(7) The 12th Japan-Taiwan Joint Workshop for Young Scholars in Applied Mathematics, Spring 2022. Organizers: Chiun-Chuan Chen (NTU) et al

(8) Chun-Hsiung Hsia plans to invite Professor Takaaki Nishida to visit NCTS for one month starting from Oct 1st, 2022.

2.5 Scientific Computing

2.5.1 Core Members

1. Program Committee Members: Tsung-Ming Huang (NTNU, chair), Wei-Fan Hu (NCU), Matthew M. Lin (NCKU), Ming-Cheng Shiue (NCTU), Suh-Yuh Yang (NCU)

2. Affiliated Faculty Members: Meng-Ho Chen (CCU), Pengwen Chen (NCHU), Ray-Bing Chen (NCKU), Chien-Hong Cho (CCU), Chia-Chieh Jay Chu (NTHU), Feng-Jui Hsieh (NTNU), Po-Wen Hsieh (NCHU), Chieh-Sen Huang (NSYSU), Feng-Nan Hwang (NCU), Yueh-Cheng Kuo (NUK), Ming-Chih Lai (NCTU), Tsung-Lin Lee (NSYSU), Jephian Chin-Hung Lin (NSYSU), Te-Sheng Lin (NCTU), Wen-Wei Lin (NCTU), Ching-Sung Liu (NUK), Ruey-Ling Sheu (NCKU), Yu-Chen Shu (NCKU), Yu-Hau Tseng (NUK), Weichung Wang (NTU), Chin-Tien Wu (NCTU), Mei-Heng Yueh (NTNU)

2.5.2 Program Overview

The NCTS Topical Program Scientific Computing is an interdisciplinary field that involves mathematical theories, computational algorithms, and domain knowledge. The program aims to develop new tools to expand current science, engineering, and applications. Its research directions are based not only on our members' strengths but also on new trends in scientific computations. These directions include matrix computations in the study of Maxwell equations and discrete Laplacians, numerical PDEs and computational fluid dynamics, mathematical problems in image processing, low-rank approximations, phase retrieval, and artificial intelligence in geometric information and medical image.

2.5.3 Research Highlights

Our group members focus on mission-oriented studies which can discover, understand, and enhance scientific principles and computing techniques toward complex and dynamic real-world problems. So far, our newly developed technologies have made a significant impact on the topics including numerical PDEs for fluid dynamics, matrix computations for nonlinear matrix equations and three-dimensional (3D) Maxwell equations, low-rank approximation, computational conformal geometric, high performance computing and image processing. We list our major contributions as follows.

1. Nonlinear matrix equations and continuation methods, by Yueh-Cheng Kuo:

   (a) We consider the nonlinear matrix equations (NME) $X + BX^{-1}A = Q$, where A, B, and Q are squire matrices. When $B = A^{\top}$, Q is complex symmetric, and the imaginary part of Q is positive definite, the NME has a unique complex symmetric stabilizing solution with a positive definite imaginary part. The assumption is satisfied for some applications including Nano Research and high-speed trains. In particular, the stabilizing solution is the solution of practical interest. The research results of this topic can be summarized as follows:

      - We provide an elementary proof of the existence of the NME and develop an efficient algorithm to compute the stabilizing solution.
      - We constructed a nonlinear differential equation of matrix pairs that are invariant in the class of symplectic matrix pairs. The solution of this nonlinear differential equation passes the iterates generated by the fixed-point iteration, the SDA, and Newton's method with some additional conditions.
      - We solve the NME that arises from the vibration analysis of high-speed trains.
      - We develop a numerical method for computing the symplectic matrix pair (M, L) which represents Hamiltonian matrix exponential.

   (b) We mainly employ the continuous method to study the solutions of a system of nonlinear equations with parameters. Under the continuous change of the parameter, we investigate the change of the solution by the continuation method. The continuation method has two advantages: 1. It can observe the continuous change of solution by changing the parameter. 2. The unstable solutions can be tracked by the continuation method. Recently, we have had some results including

      - We compute the Z-/H-eigenpairs of nonnegative tensors by continuation methods.
      - Computing the CP decomposition of an unbalanced tensor.
      - Find the best rank-1 approximation of nonnegative tensors.

2. Numerical PDEs and computational fluid dynamics, by Ming-Chih Lai:

   Recently, we have started the simulations of Newtonian vesicle in Oldroyd-B fluid under shear flow. It is surprising to find that the stationary inclination angle can be negative without the transition to tumbling (TB) motion. Moreover, the inertia effect plays a significant role that is able to turn the vesicle back to positive inclination angle through TT-TB-TT transition as the Reynolds number

increases. To the best of our knowledge, this is the first numerical work for the detailed investigations of Newtonian vesicle dynamics suspended in viscoelastic Oldroyd-B fluid. Using the developed skew-adjoint property in our previous approach, we have successfully developed a linearly semi-implicit scheme for the inextensible interface with bending in unsteady Stokes flow and proved the scheme is unconditionally energy stable. Based on the unconditionally energy stable scheme, we are able to develop an Immersed Boundary Projection Method (IBPM) to efficiently solve the problem in a fractional-step manner. Both fluid incompressibility and interface inextensibility can be satisfied simultaneously. We also have coupled different methods such as Grid Based Particle Method (GBPM) and Implicit Boundary Integral Method to study 3D insoluble surfactant problem, and GBPM and Immersed Interface Method to solve 3D electrohydrodynamic problem. Our results show the coupled methods significant efficacy for simulating 3D interfacial flow problems. Meanwhile, we have developed a novel numerical method that redistributes unevenly given points on an evolving closed curve to satisfy equi-arclength condition. We demonstrate its practical applicability by showing an evolving curve with large deformation in various flows such as mean curvature flow, Willmore flow, and Stokes flow.

3. Inverse eigenvalue problems and low rank approximation, by Matthew M. Lin:

   We, in this study, propose an eclectic mix of methods from differential geometry and scientific computing to reconstruct a structured matrix, for example, nonnegative entries, fixed diagonal entries, or even predetermined arbitrary entries. This work can be regarded as a numerical method to realize the so-called Weyl-Horn condition, which presents the relations between the eigenvalues and singular values of an arbitrary matrix. In theory, we show that the proposed inexact Newton method not only converges globally and quadratically but also calculates the desired solution robustly and efficiently.

   Low rank approximation plays an essential role in data analytics. Its goal is to analyze a given data and reveal the embedded information. The works on this topic include the analysis of integer datasets and the qualification of entanglement among entangled bipartite systems. Two results are explained as follows:

   (a) Integer data analytics: To analyze integer datasets, we, in this work, proposes a way of calculating integer matrix approximation while keeping the original dataset characteristics. This method is based on an integer least squares estimation, where a global optimization is obtained in each iteration. Numerical applications include but are not limited to studies of association rule mining, cluster analysis, and pattern extraction. Numerical experiments suggest that our method can calculate a more accurate solution for analyzing discrete datasets than other conventional methods.

(b) Entanglement qualification: Gauging the distance between a mixed state and the convex set of separable states in a bipartite quantum mechanical system over the complex field is a tough problem with many applications in quantum informatics. As a first step toward this challenging problem, we investigate the rank-1 approximation of a bipartite system over the real field where the entanglement happens. We show that this approximation can be recast as a nonlinear eigenvalue problem or a nonlinear singular value problem. Two iterative methods with rigorously convergence analysis are given correspondingly. We believe our contributions can serve as a building block for the more complicated multipartite systems or higher-rank approximation problems.

4. Maxwell's equations for three-dimensional photonic crystals, led by Wen-Wei Lin:

   This topic focuses on studying the eigenstructure of generalized eigenvalue problems (GEPs) arising in the three-dimensional (3D) source-free Maxwell's equations for photonic crystals and bi-anisotropic complex media with 3-by-3 permittivity tensor $\varepsilon > 0$, permeability tensor $\mu > 0$, and scalar magnetoelectric coupling constants $\xi = \bar{\zeta} = \imath\gamma$.

   For the photonic crystals, we propose the Fast Algorithms for Maxwell's Equations (FAME) package for solving Maxwell's equations for modeling three-dimensional photonic crystals. FAME combines the null-space free method with fast Fourier transform (FFT)-based matrix-vector multiplications to solve the generalized eigenvalue problems (GEPs) arising from Yee's discretization. The GEPs are transformed into a null-space free standard eigenvalue problem with a Hermitian positive-definite coefficient matrix. We successfully use FAME on a single P100 GPU to solve a set of GEPs with matrices of dimension more than 19 million, in 127 to 191 seconds per problem. These results demonstrate the potential of our proposed package to enable large-scale numerical simulations for novel physical discoveries and engineering applications of photonic crystals.

   The bi-Lebedev scheme is applying to discretize the Maxwell's equations for the bi-anisotropic complex media. The resulting GEP has eigenvalues appearing in quadruples $\{\pm\omega, \pm\bar{\omega}\}$. We consider two main scenarios, where $\gamma < \gamma_*$ and $\gamma > \gamma_*$ with γ_* being a critical value. In the former case, all the eigenvalues are real. In the latter case, the GEP has complex eigenvalues, and we particularly focus on the bifurcation of the eigenstructure of the GEPs. Numerical results demonstrate that the newborn ground state has occurred after $\gamma = \tilde{\gamma} > \gamma_*$, and the associated eigenvector has an exotic phenomenon of localization. Moreover, the Poynting vectors of the newborn eigenvector are not only concentrated in the material but also display exciting patterns.

5. Computational conformal geometric, led by Wen-Wei Lin and S.-T. Yau:

   Surface parameterizations have been widely applied to digital geometry processing. In 2017, we propose an efficient conformal energy minimization (CEM) algorithm for computing conformal parameterizations of simply-connected open surfaces with a very small angular distortion and a highly improved computational efficiency. In addition, we generalize the proposed CEM algorithm to computing conformal parameterizations of multiply-connected surfaces. Furthermore, we prove the existence of a nontrivial accumulation point of the proposed CEM algorithm under some mild conditions. Several numerical results show the efficiency and robustness of the CEM algorithm comparing to the existing state of the art algorithms. An application of the CEM on the surface morphing between simply-connected open surfaces is demonstrated thereafter. Thanks to the CEM algorithm, the whole computations for the surface morphing can be performed efficiently and robustly.

   Next, we propose a novel stretch energy minimization (SEM) algorithm for the computation of equiareal parameterizations of simply connected open surfaces with very small area distortions and highly improved computational efficiencies. In addition, the existence of nontrivial limit points of the SEM algorithm is guaranteed under some mild assumptions of the mesh quality. Numerical experiments indicate that the accuracy, effectiveness, and robustness of the proposed SEM algorithm outperform the other state-of-the-art algorithms. Applications of the SEM on surface remeshing, registration, and morphing for simply connected open surfaces are demonstrated thereafter. Thanks to the SEM algorithm, the computation for these applications can be carried out efficiently and reliably.

   Manifold parameterizations have been applied to various fields of commercial industries. In 2019, we develop a novel volumetric stretch energy minimization algorithm for volume-preserving parameterizations of simply connected 3-manifolds with a single boundary under the restriction that the boundary is a spherical area-preserving mapping. In addition, our algorithm can also be applied to compute spherical angle- and area-preserving parameterizations of genus-zero closed surfaces, respectively. Several numerical experiments indicate that the developed algorithms are more efficient and reliable compared to other existing algorithms. Numerical results on applications of the manifold partition and the mesh processing for three-dimensional printing are demonstrated thereafter to show the robustness of the proposed algorithm.

   However, it is still challenging when the topology of manifolds is nontrivial, e.g., the 3-manifold of a topological solid torus. In 2020, we propose a novel volumetric stretch energy minimization algorithm for volume-preserving parameterizations of toroidal polyhedra with a single boundary being mapped to a stan-

dard torus. In addition, the algorithm can also be used to compute the equiareal mapping between a genus-one closed surface and the standard torus. Numerical experiments indicate that the developed algorithm is effective and performs well on the bijectivity of the mapping. Applications on manifold registrations and partitions are demonstrated to show the robustness of our algorithms.

6. Analysis and computation of the geophysical fluid dynamics, by Ming-Cheng Shiue

   (a) Long time stable numerical schemes and time periodic solutions.
   We consider a semi-discretized Euler scheme to solve the three dimensional viscous primitive equations. Based on suitable assumptions on the initial data and forcing terms, the long-time stability of the proposed scheme is proven by showing that the H1 norm (in space variables) of the solutions is bounded at each time step when the time step satisfies certain smallness condition. This result matches the continuous case. Meanwhile, we consider several models including Primitive equations of large-scale ocean, atmosphere with moisture and two-dimensional Navier-Stokes equations with time-periodic forcing terms. The asymptotic stability criterion and the existence of the solution are proven under the smallness assumption on the nontrivial forcing terms.

   (b) Porous media flow modelled by Navier-Stokes equations coupling with Darcy flow.
   We performed numerical analysis of MAC scheme for Stokes and Darcy coupling flows and also gave a simple projection method for Navier-Stokes and Darcy coupling flow with curvy interface.

7. Mathematical problems in image processing, by Suh-Yuh Yang:

   We have proposed a novel adaptive variational model for contrast enhancement for partially shaded low-light images in the paper [SIAM Journal on Imaging Sciences, 13 (2020), pp. 1-28.]. The contrast enhancement plays an important role in image/video processing and computer vision applications. Its primary purpose is to adjust the image intensity to enhance the quality and features of the image. In this paper, we have proposed a simple and efficient adaptive variational model for contrast enhancement for partially shaded low-light images. The key idea of this adaptive approach is to employ the maximum image of the color RGB channels as a classifier to divide the image domain into the relatively bright and dim parts. Then different fitting terms are used for each part such that the bright pixels are preserved as close as possible to the original ones while the dim pixels are boosted with brightness and contrast-level parameters to adjust the degree of the strength. With this adaptivity, one can find that the proposed

model considerably improves the existing variational models in the literature. In this work, the existence and uniqueness of minimizer for the variational minimization problem are established. The split Bregman method is used to accomplish an efficient numerical implementation of the adaptive variational model. Moreover, a number of numerical experiments and comparisons with other popular enhancement methods are conducted to demonstrate the high performance of the newly proposed method. The idea used in the proposed adaptive variational model can be extended to other image processing problems such as image dehazing, color correction, and image stitching, etc.

8. Multi-scale flows of compressible Euler Equations, by Chia-Chieh Jay Chu:

   We use multi-scale expansions to study the compressible Euler equations near-vacuum states. The vacuum states of the compressible Euler equations have been investigated for decades by many researchers. In our work, we propose a different model to simulate the situation when initial density is very close to 0 and velocity is $\mathcal{O}(1)$. We expand the density and the velocity according to the amplitudes and derive the leading-order system for the leading-order term in the expansion. We explore the properties of the solutions of the leading-order system and prove the existence of vacuum states in the multi-scale gas flows governed by the Cauchy problem of compressible Euler equations.

9. Runge-Kutta WENO scheme for advection-diffusion equations, by Chieh-Sen Huang:

   He and his colleagues developed both explicit and implicit finite volume weighted essentially non-oscillatory (WENO) schemes in multiple space dimensions on non-uniform computational meshes. The diffusion degeneracy is reformulated through the use of the Kirchhoff transformation. Space is discretized using WENO reconstructions with adaptive order (WENO-AO), which have several advantages, including the avoidance of negative linear weights and the ability to handle irregular computational meshes. A special two-stage WENO reconstruction procedure is developed to handle degenerate diffusion. Element averages of the solution are first reconstructed to give point values of the solution, and these point values are in turn used to reconstruct the Kirchhoff transform variable of the diffusive flux. Time is discretized using the method of lines and a Runge-Kutta time integrator. Strong Stability Preserving (SSP) Runge-Kutta methods are used for the explicit schemes, which have a severe parabolically scaled time step restriction to maintain stability. They also develop implicit Runge-Kutta methods. They present in detail schemes that are third order in both space and time in one and two space dimensions using non-uniform meshes of intervals or quadrilaterals. Through a von Neumann (or Fourier mode) stability analysis, we show that smooth solutions to the linear problem are unconditionally L-stable on

uniform computational meshes when using an implicit Radau IIA Runge-Kutta method.

2.5.4 Highlights of Events

1. 2021 NCTS Undergraduate Summer Research Program:
   (i) Group 7: Applications of Deep Learning for Scientific Computing
   Supervisor: Wei-Fan Hu (NCU)
   Summary: In the mathematical theory of artificial neural networks, the universal approximation theorem illustrates the ability of artificial neural networks to approximate arbitrary functions. In 1989, G. Cybenko proposed a general approximation theorem using a single hidden layer as the architecture and the sigmoid function as the activation function. After that, some researchers have put forward the approximate theorem of generalized neural network architecture, which gives us enough confidence to apply this method to solve problems in scientific computing. This project mainly focus on learning the architecture and approximation methods of neural networks and how to complete the training through backpropagation.

   We use this method to solve simple one-dimensional Poisson equations or other differential equations and compare with the advantages and disadvantages of traditional numerical methods.

   This project is divided into two parts, namely the pre-course and program implementation. The course content includes the following topics:

   - Feedforward neural network architecture and mathematical model
   - Examples and applications
   - Optimization problem-the definition of loss function
   - Neural network training-gradient descent method and stochastic gradient method
   - Backpropagation
   - Numerical methods of differential equations

   After learning the relevant background knowledge, students have group discussions. The following relevant topics are discussed:

   - The relationship between neural network-like architecture and continuous function fitting
   - Fitting method of non-differentiable function and discontinuous function

- Use deep learning methods to solve one-dimensional Poisson problems or ordinary differential equation problems

(ii) Group 8: Optical phase retrieval problem
Supervisor: Pengwen Chen (NCHU)
Summary: In signal processing or image processing, people aim to reconstruct a signal/image from various measured data. In this project, we are interested in the recovery from underdetermined measurements. For instance, the standard compressive sensing problem consists in the reconstruction of a sparse vector or a low-rank matrix. Recently, many convex-relaxation based algorithms as well as non-convex algorithms are proposed to solve an optics inverse problem–phase retrieval. In this project, students are asked to test a few algorithms in this field and work on some research topics. We provide the background knowledge.

- Sparse model and low-rank matrix model.
- Basic algorithms for sparse model and low-rank matrix model: Thresholding-based algorithms and restricted isometry property.
- Popular convex algorithms for L1-norm/Nuclear norm minimization problems.
- Conventional algorithms in phase retrieval.
- Radon transform and X-ray transform.

2. 2021 South Taiwan Workshop on Scientific Computing, Differential Equations and Applications, January 29, 2021, National Pingtung University:

3. Opportunities and Challenges in Numerical Algebra (I)-(VI), 2021, 1/27, 2/3, 2/17, 2/24, 3/20, 3/27 (online workshop):
Numerical algebra has been playing pivotal roles as the pillar of scientific computing for decades. Theories and numerical techniques to its core problems including linear systems, least squares problems, and eigenvalue problems are taken very much for granted and used daily in science and engineering without questioning. Today numerical algebra sits at the cross road of reinventing itself to adapt to a rapidly changing landscape because of the emergence of data science and artificial intelligence, while its original core mission remains. This online workshop series consists of a series of advanced technical talks on latest developments in numerical algebra and its recent machine learning applications.

4. 2021 Conference on Advanced Topics and Auto Tuning in High-Performance Scientific Computing, March 19-20, 2021, National Central University:

The Conference on Advanced Topics and Auto Tuning in High-Performance Scientific Computing focuses on the scientific impacts due to the latest computer architectures and the approaches to achieve high-performance computing on these leading-edge computers. Advances in many-core architectures and high-end computers have unveiled their significances in the scientific discoveries and engineering achievements. The complexity of these newly developed computers, however, also leads to contemporary challenges to achieve the best efficiency of the highly promising computational capabilities. The conference encourages interdisciplinary communications between researchers from applied mathematics, statistics, computer science, physical sciences, engineering and industry to prompt innovations and breakthroughs in this exciting field. The main themes include, but not limited to, simulations, numerical methods, applications, hardware, and particularly software and algorithm auto-tuning via statistical methods.

5. NCTS Student Workshop on Scientific Computing, April 24, 2021, National Yang Ming Chiao Tung University:
   The aim of this workshop is to enable young researchers and graduate students in the field of scientific computing and related disciplines to better understand recent studies in different fields.

6. NCTS (Central Taiwan) computational mathematics workshop, April 24, 2021, National Chung Hsing University:
   This workshop aims to provide one academic discussion platform on the theory and application of computational mathematics. The focus is on the application of various Arnoldi algorithms, including circuit simulations. Topics include matrix exponentials, model order reduction and quadratic eigenvalue problem.

7. Taiwan Mathematics School: Mathematical foundation toward artificial intelligence in medicine, 2021-02-23 / 2021-06-17, Wu Hau-Tieng (Duke University).

8. Machine learning on solving partial differential equations (Online Conference), Sep 2, 2021
   For the past decade, deep learning has achieved great success in different scientific disciplines, including image recognition, natural language processing, and many other practical applications in our daily life. The main question for the Math community is how the AI can help us do our research. In this special lecture, we give an example of using machine learning methods to solve partial differential equations, especially when PDEs become difficult to handle with traditional numerical methods (such as finite difference, finite element or spectral methods). We start with our development of machine learning methodology by considering elliptic interface problems where the solutions are usually not

smooth. We have successfully developed two different types of completely shallow neural network to solve above elliptic PDEs. To the best of our knowledge, this is the first work in literature that uses only one hidden layer with moderate number of neurons to solve the problems even in higher dimensions. We start by introducing some fundamental mathematical backgrounds for machine learning and the methodologies to solve PDEs. Then, we introduce our shallow neural network model to solve the elliptic interface problems in detail. Some remaining open questions and future directions are also raised.

9. 2021 NCTS Workshop on Computational Mathematics and Scientific Computing for Young Researchers, September 10-11, 2021 (online workshop).
   The purpose of this workshop is to provide scholars with an opportunity to share their research and interests in the field of computational mathematics and related topics. In particular, senior scholars not only present their research, but also share their experiences. It is the hope that this workshop can inspire more collaboration and interaction between senior and young scholars in the future.

2.5.5 Future plans

1. 2022 South Taiwan Workshop on Scientific Computing, Differential Equations and Applications

2. Conference on Advanced Topics and Auto Tuning in High-Performance Scientific Computing, 2022, March

3. 2022 NCTS Student Workshop on Scientific Computing, April, Fu Jen Catholic University

4. NCTS/NCU Seminar on Computational and Data Sciences
   Organizer: Suh-Yuh Yang (NCU)
   Time: This seminar will be held bi-weekly on Monday, 03:00-05:00 pm
   Venue: Room 430, Hong-Jing Hall, NCU
   Theme: Computational and data science is an important interdisciplinary research field among statistical science, computer science and mathematics. Specifically, compressive sensing is an optimization technique to efficiently acquire or reconstruct sparse information from few measurements. It has been applied successfully in signal/image processing, data analysis, medical imaging, machine learning, etc. We organize this regular seminar to pursuit possible applications of compressive sensing in the following areas: (i) sparse dictionary learning, (ii) image processing, and (iii) numerical methods for multi-scale partial differential equations.

5. Seminar on Ice sheet dynamics, Organizer: Ming-Cheng Shiue

6. Professor Ming-Cheng Shiue will invite Professor Saito Norikazo from the University of Tokyo and his team Professor Takahito Kashiwabara from the University of Tokyo, and Professor Issei Oikawa from the University of Tsukuba for mutual academic visits and cooperation.

7. Professor Wei-Fan Hu will invite Professor Chaouqi Misbah from Université Grenoble Alpes in France to come to Taiwan for an academic visit.

2.6 Interdisciplinary Studies

2.6.1 Core members

1. Program Committee Members: Je-Chiang Tsai (NTHU, chair), Te-Sheng Lin (NYCU), Hau-Tieng Wu (Duke), Yng-Ing Lee (NTU), Shun-Jen Cheng (AS).

2. Affiliated Faculty Members: Jein-Shan Chen (NTNU), Ray-Bing Chen (NCKU), Chang-Yuan Cheng (NPTU), Chia-Chieh Jay Chu (NTHU), Mei-Hui Guo (NSYSU), Tzyy-Leng Horng (FCU), Chih-Hao Hsieh (NTU), Chao-Ping Hsu (AS), Tai-Chia Lin (NTU), Yu-Lun Lo (CGMH), Hung-Chi Kuo (NTU), Chi-Jen Wang (CCU), Feng-Bin Wang (CGU), Weichung Wang (NTU), Tsung-Fang Wu (NUK).

3. Postdoctoral Fellows: Chun-Wei Chang (NCTS).

2.6.2 Program Overview

The purpose of the Topical Program Interdisciplinary Studies at the NCTS is to assist domestic mathematicians whose research expertises are combinations of mathematics and other academic disciplines to develop scientific topics, which in turn stimulate the birth of new mathematical theory or problems. Our research topics are motivated by various scientific processes and phenomena. In the past year, our members focus on the following areas:

a. Optimization and big data: Jein-Shan Chen.

b. Mathematical ecology: Chang-Yuan Cheng and Feng-Bin Wang.

c. Mathematical modeling and analysis of ion channels: T.-L. Horng and T.-C. Lin.

d. Time series analysis for nonlinear dynamical systems: Chih-Hao Hsieh and Chun-Wei Chang.

e. Mathematical modelling on fluid dynamics: Te-Sheng Lin.

f. Complex network systems: Je-Chiang Tsai.

g. Medical signal processing: Hau-Tieng Wu.

2.6.3 Research Highlights

a. Optimization and big data, *by J.-S. Chen.*

We propose a new smoothing strategy along with conjugate gradient algorithm for the signal reconstruction problem [1]. Theoretically, the proposed conjugate gradient algorithm along with the smoothing functions for the absolute value function is shown to possess some nice properties which guarantee global convergence. Numerical experiments and comparisons suggest that the proposed algorithm is an efficient approach for sparse recovery. Moreover, we demonstrate that the approach has some advantages over some existing solvers for the signal reconstruction problem.

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1. *Smoothing strategy along with conjugate gradient algorithm for signal reconstruction*, J. Sci. Comput., **87** (2021), 21.

b. Mathematical ecology, *by C.-Y. Cheng and F.-B. Wang.*

1. Effect of spatial heterogeneity on virus dynamics

The well-mixed virus models are based on rapid blood circulation, and regard the within-host viral environment as a spatially homogeneous compartment. However, the spatial heterogeneity arising from specific organs, such as the brain, testes, spleen, and lymph nodes, can result in different levels of infection within the host or inconsistent drug concentrations during treatment. Accordingly, we explore this subject from models in either discrete or continuous space. For the discrete case in [1], we showed that viral infection may be underestimated if a well-mixed model is used, and the optimal drug administration depends not only on the drug distribution over various compartments but also on the timing of the administration of different drugs. In [2] considering the age sine infection, we further showed that the effect of migration rate of cells between organs may be non-monotone. We further explored the continuous case in [3] via a reaction-diffusion model, based on a non-local infection, to describe productively infected cells and viruses that survived the immature stage. In addition to theoretically investigating the extinction or persistence of virus, we numerically showed that the dependence of the threshold value on the intracellular delay could be non-monotone, depending on the death rate

of infected cells in the immature stage, and the spatial fragmentation of the virus environment enhanced viral infection.

2. Dengue transmission model in time-space periodic environment

Dengue is a vector-borne infectious disease which is transmitted to humans mainly by the bites of Aedes aegypti mosquitoes. In order to investigate the spreading dynamics of Aedes mosquitoes, the authors in [4] proposed an advection-dispersion-reaction equation coupled with an ordinary equation, in which populations are divided into two sub-populations, the winged/mature female mosquitoes and the aquatic population. The study in [5] is devoted to further investigation of dengue spread via a time-space periodic reaction-advection-diffusion model, which is an extended version of the one in [4]. We establish the existence of the spreading speeds and its coincidence with the minimal speed of almost pulsating waves.

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c. Mathematical modeling and analysis of ion channels, *by T.-L. Horng and T.-C. Lin.*

We investigate the electric discharge of electrocytes by extending our previous work on the generation of electric potential. We first give a complete formulation of a single cell unit consisting of an electrocyte and a resistor, based on a Poisson-Nernst-Planck (PNP) system with various membrane currents as interfacial conditions for the electrocyte and a Maxwell's model for the resistor. Using asymptotic analysis, we simplify our PNP system and reduce it to an ordinary differential equation (ODE) based model. Unlike the case of an infinite resistor, our numerical simulations of the new model reveal several distinct features. A finite current is generated, which leads to non-constant electric potentials in the bulk of intracellular and extracellular regions. Furthermore, the current induces an additional action potential

(AP) at the non-innervated membrane, contrary to the case of an open circuit where an AP is generated only at the innervated membrane. The voltage drop inside the electrocyte is caused by an internal resistance due to mobile ions. We show that our single cell model can be used as the basis for a system with stacked electrocytes and the total current during the discharge of an electric eel can be estimated by using our model.

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d. Time series analysis for nonlinear dynamical systems, *C.-H. Hsieh and C.-W. Chang.*

1. Identifying causality in nonlinear dynamical systems based on various time series analysis methods.

Empirical mode decomposition (EMD) [1] is a well-established approach to decompose signals from time series data into a superposition of intrinsic mode functions (IMFs); Yang et al. [2] proposed a novel application of EMD for detecting causation in general dynamical systems. The casual inference is based on the criterion that time series coherence is diminished when the causal-related component is removed from the effect. However, the proposed approach fails to correctly identify causal relationships for a system of two independent variables driven by a shared external forcing and is not superior to the existing method (Convergent Cross Mapping, CCM [3]) in detecting causality in nonlinear dynamical systems. Although the proposed EMD method effectively detects causality in systems comprised of superposing signals, caution is needed when applying this approach in more general dynamical systems. In this project, we aim to showed that the EMD method proposed in Yang et al [2] fails to correctly identify causal relationships for a system of two independent variables driven by a shared external forcing (aka Moran effect). In addition, we provided clear evidence that the Moran effect can be distinguished from true causality using the methods developed based on Takens Theorem. To summarize, the EMD method clearly works for systems in which superposition is obtained, but does not provide unambiguous results for non-separable, nonlinear dynamical systems.

2. Reconstructing large interaction networks from empirical time series data.

Reconstructing interactions from observational data is a critical need for investigating natural biological networks [4], wherein network dimensionality is usually high [5]. However, these pose a challenge to existing methods that can quantify only small interaction networks. Here, we proposed a novel approach [6] to reconstruct high-dimensional interaction Jacobian networks using empirical time series without

specific model assumptions. This method, named ?multiview distance regularized S-map,? generalized the state space reconstruction to accommodate high dimensionality and overcome difficulties in quantifying massive interactions with limited data. When evaluating this method using time series generated from theoretical models involving hundreds of interacting species, estimated strengths of interaction Jacobians were in good agreement with theoretical expectations. Applying this method to a natural bacterial community helped identify important species from the interaction network and revealed mechanisms governing the dynamical stability of a bacterial community [7]. The proposed method overcame the challenge of high dimensionality in large natural dynamical systems.

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e. Mathematical modelling on fluid dynamics, *by Te-Sheng Lin.*

1. Instabilities of a thin liquid film in a funnel.

Thin liquid films with fronts involving contact lines and their instabilities are relevant to applications in a number of different fields, ranging from nanoscale to macroscale films where instabilities are driven by a combination of various body forces, surface tension, and wettability, see [1, 2] for reviews. Significant progress has been reached by using long wave approximation, which simplifies considerably the analysis of thin film flows and their stability. In the context of thin films on a

macroscale, the setup involving completely wetting film of constant thickness flowing down an incline has been understood reasonably well. For such configuration, linear stability analysis carried out in a moving reference frame leads to the dispersion relation which shows stability for large wavenumbers, and predicts the most unstable wavelength (specifying the distance between emerging fingers), which results from the balance between destabilizing gravity and stabilizing surface tension forces. However, as soon as some of the simplifying assumptions are removed, understanding the instability becomes much more complicated.

We explore flow of a completely wetting fluid in a funnel, with particular focus on contact line instabilities at the fluid front [3]. While the flow in a funnel may be related to a number of other flow configurations as limiting cases, understanding its stability is complicated due to the presence of additional azimuthal curvature, as well as due to convergent flow effects imposed by the geometry. Convergent nature of the flow leads to thickening of the film, therefore influencing its stability properties [4]. In this work, we analyze these stability properties by combining physical experiments, asymptotic modeling, self-similar type of analysis and numerical simulations. We show that a reasonably complete understanding of instability development for a film flowing in a funnel can be reached by combining the insight from experiments and asymptotic analysis that allows for significant simplification of the governing equations. Furthermore, it turns out that despite the complexity of the problem, a useful insight can be also reached by considering a self-similar approach similar to the one used for the flow down an incline plane. Such insight from self-similar methods combined with linear stability analysis originated from the flow down an incline provides an important guidance in carrying out numerical simulations that help to develop better understanding of the instability development. While we have focused on a particular geometry of flow in a funnel, we note that a similar approach could be applied to a number of other unstable flows, such as the flows on a sphere, outside surface of a funnel, or even in more complicated geometries.

We note that instabilities of the systems whose base state evolves in time are difficult to analyze in a tractable manner. For the present problem, we have shown that a reasonably good insight can be reached by simplifying the problem first, and then using some input regarding instability development from the experiments. One would of course like to be able to understand the general features of instability development, including the factors that govern instability onset itself. Reaching this goal will require further development of stability analysis and is left as an open problem for future work.

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f. Complex network systems, *by J.-C. Tsai.*

1. Spreading phenoma in the Neolithic transition in Europe.

The Neolithic transition began the spread of early agriculture throughout Europe through interactions between farmers and hunter-gatherers about 10,000 years ago. Archeological evidence produced by radiocarbon dating indicates that the expanding velocity of farming is roughly constant all over Europe. However, the expanding velocity determined by existing modeling approaches is faster than the observed velocity [1]. For understanding this difference, we propose a three-component reaction-diffusion system which consists of two different types of farmers (sedentary and migratory) and hunter-gatherers from the viewpoint of the influence of farming technology. Our purpose is to study the relation between the expanding velocity of farmers and the farming technology parameter. The propagating velocity derived from our model can be compatible with the observed velocity when farming technology is developed. Our results suggest that the reason for the slowdown of the Neolithic transition might be related to the increase in the development of farming technology. Moreover, the bifurcation structure of waves in our proposed model was investigated [2]. The results indicate that there is rich structure of traveling waves in the model. Finally, we also compute the spreading speed of the model when the farmer population is initially with localized support and the hunter-gatherer is uniformly distributed [3].

2. Structural bifurcation analysis in chemical reaction networks.

In living cells, chemical reactions form complex networks, and thus give a very large ODE system. Dynamics arising from such networks are the origins of biological functions. Also the crucial processes are easily obscured by numerical approaches due to the complexity of the underlying networks. We propose an analytical approach, not numerical one, to analyze bifurcation behaviors of network systems using their structures alone [4]. This approach gives an efficient analytical way to determine the bifurcation point, and a satisfactory understanding of the essential mechanism underlying the biological process. In particular, it might be employed to predict the unknown pathways in the chemical networks or biological

processes. We recently have established the spectrum correspondence between our setting and the classical one [5]. Finally, the framework of this approach has been applied to our recent experimental work of the E2F6 regulation on miR-193a in the ovarian carcinogenesis [6].

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g. Medical signal processing, *by H.-T. Wu.*

We developed a novel nonstationary long-term and high frequency biomedical signal analysis tool by taking into account the manifold learning tools. The algorithm is based on a preprocessing of the biomedical signal and the unsupervised learning algorithm, diffusion maps, which we call the "dynamic diffusion map (DDmap)". It is applied to electrocardiogram and arterial blood pressure signal analysis and now applied to clinical applications. We also develop a series of theoretical supports to the DDmap algorithm, ranging from the spectral convergence and its robustness to high dimensional noise behavior.

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2.6.4 Highlights of Events

We run this program by means of regular seminars and summer courses, as well as workshops and conferences. We briefly describe them below.

2.6.4.1 Seminars

We organize regular seminars to facilitate research discussions and help students broaden their horizons. Seminars on optimization are organized by J.-S. Chen in Taipei. Seminars on applied mathematical analysis are run by C.-C. Chu, T.-S. Lin and J.-C. Tsai in Hsinchu. Seminars on mathematical biology are organized by C.-Y. Chang and F.-B. Wang in Hsinchu. Online seminar on gene network analysis are organized by W.-Y. Chan and J.-C. Tsai.

2.6.4.2 Workshops and Conferences

During this past year, we have held several academic activities at the NCTS. In particular, Prof. Hau-Tieng Wu has organized workshops which facilitate discussion between physicians, mathematicians, and statisticians. We look forward to having more research cooperations between mathematical and other scientific communities. The following is a list of workshops and conferences we hosted during the past year.

a. Meet physicians, mathematicians and statistcians, Mar. 20, 2021. Organizer: Hau-Tieng Wu.

b. NCTS Symposium on Applied Mathematics, Jun. 28, 2021. Organizers: C.-C. Chu, T.-S. Lin, and J.-C. Tsai.

2.6.5 Future Plans

In order to broaden and deepen our study on current projects, we plan to invite our long-term collaborators such as

a. Carla Bosia (Italian Institute for Genomic Medicine, Italy),

b. Jun Kitagawa (MSU, USA),

c. Chun Liu (Illinois Institute of Technology, USA),

d. Atsushi Mochizuki (Kyoto university, Japan),

e. Marc Pradas (Open University, UK),

f. Stefan Steinerberger (Washington University, USA),

g. Ronen Talmon (Technion, Isreal),

h. Uwe Thiele (University of Munster, Germany),

and others. During their visit, we will hold workshops or conferences to provide a platform for research discussion between foreign visitors and local researchers. The following is the list of the proposed workshop or conference.

2.6.5.1 Workshops and Conferences

a. Mini-symposium on Applied Mathematics for Students, Jan., 2022.

b. Meet physicians, mathematicians and statisticians, part 3, Jan., 2022.

c. Modeling and analysis in molecular biology and electrophysiology, Jan., 2022.

2.6.5.2 Courses and Lectures

In the upcoming year, there will be two courses. One is the Taiwan Mathematical School on Continuation method and its applications. The lecturers are Yueh-Cheng Kuo from National University of Kaohsiung, Te-Sheng Lin from National Yang Ming Chiao Tung University, Chia-Chieh Jay Chu and Je-Chiang Tsai from National Tsing Hua University. They will explain the theory of continuation methods, its variants, and its applications to other subjects such as fluid dynamics and reaction-diffusion systems. The other course is the course on dynamical systems and its application to mathematical ecology. Below is the list of courses in the coming academic year.

a. Taiwan Mathematical School: Continuation Method: Theory and Application, Sep., 2021 - Jan., 2022. Organizers: C.-C. Chu, Y.-C. Kuo, T.-S. Lin, and J.-C. Tsai.

b. NCTS Summer Course on Mathematical Biology, Aug., 2022.

2.6.5.3 Seminars

We continue to hold regular seminars to facilitate research discussions and help students broaden their horizons. Seminars on optimization are organized by J.-S. Chen in Taipei. Seminars on applied and industrial mathematics analysis will focus on problems from industry. Seminars on mathematical biology will put attention on model construction and analysis. Seminars on gene network analysis will concentrate on how noises propagate in large network systems and its implications.

3 Host University Commitment

The commitment of the host institution, the National Taiwan University, consists of the following: budget, space, and logistic support. Overall, the host institution has been very supportive. Its commitment on budget has been completely fulfilled as described in the above Section **??** on budget and expenditures. Below we will focus on the part of its commitment regarding space and other logistic supports.

3.1 Space of NCTS

The Center moved to the newly-built Cosmology Building in October of 2020. The space in the new building is very nice and pleasant. According to the commitment of the host university, where the official document is attached in the following pages, NTU provides 200 ping in Cosmology Building free of charge for the Center to use. The Center also rents extra space in the building at a special and guaranteed rate. Space in parts of the second and third floor of the New Mathematics Building is also provided free of charge for the Center to use. Because it is an old building, the condition there is not perfect. We are still working on how to make best use of this space after relocation of the Center to the Cosmology Building. We continue to use some space on the fourth floor of the Astronomy-Mathematics Building provided by the Department of Mathematics, which includes one lecture room for classes/talks and two offices for the NCTS research assistants.

3.2 Logistic support of NTU

The logistic support of the NTU in recent years has been outstanding. Whenever needed, it is easy to make appointments to discuss directly with administrative officers including the President, Vice Presidents, Provost, Dean of Research and Development, Dean of General Affairs, etc. In addition to the items mentioned in the official document of commitment, Taiwan Mathematics School (TMS) also has been greatly supported by NTU. The Center can offer courses with credits, and allow students from the cooperative universities or cooperative departments of the NCTS to take the courses.

申請機構配合事項同意書

計畫名稱：國家理論科學研究中心第五階段運作計畫(2021.1.1-2025.12.31)

計畫主持人姓名/職稱：李百祺/教授

申請機構配合措施：本計畫業經單位內部審查，同意提供下列配合事項。

一、配合款：本機構同意提供科技部核定經費之40%為配合款，於執行期間優先使用於計畫所需各項經費（含中心人員薪資、學術活動費用、使用空間的場租、軟硬體設備、裝修維護費、水電雜支等等）。本計畫執行期滿後，收支報告表內需詳細註明配合款支用情形。

二、員額：

(1) 提供數學組、物理組至少各10名博士後研究員名額，以招募優秀年輕研究人員。(於配合款中編列相關薪資)

(2)中心主任與執行主任因推動中心業務，得依本校「專任教師授課時數減免要點」相關規定申請減免授課時數。

三、管理費：依本校「建教合作計畫管理費分配處理細則」辦理，以科技部計畫15%管理費計，如不分配至計畫主持人所屬學院(須先與學院議定)，則分配至校級中心之管理費約32%。

四、結餘款：依本校「建教合作計劃結餘款分配、運用及管理要點」辦理，依當年度結餘款總額扣除個別使用款項後之餘額結算。

五、空間：

(1)數學組使用現況及規劃：

專屬空間共約 549 坪

次震宇宙館(數學組)	4樓(200坪)及5樓無償使用空間	辦公室、小型會議室與研究室。
	租用空間 (數學組另行租借246.51坪)	中心行政區，小型研討室(30人)及大型研討室(120人)各1間，4間訪問學者辦公室與交誼區；走道公共空間設有沙發及茶水區，為公告區及休息討論區。辦公室、小型會議室與研究室。
數學研究中心	2樓(約66坪)	5間辦公室(供博士後研究員使用)及休息區
	3樓(約36坪)	6間訪問學者辦公室。

(2)物理組使用現況及規劃：

專屬空間共約 501 坪

次震宇宙館(物理組)	4 樓(200 坪)	11 間辦公室、小型研討室(25 人)及大型研討室（100 人）各一間，設有開放式討論空間、休息討論區、茶水區。
	租用空間(物理組另行租借 246.51 坪)	中型討論室 3 間(50 人)、小型討論室 2 間(25 人)，12 間訪客辦公室，另設有茶水區、休息討論區與公告區。
物理系	8 樓(約 54 坪)	2 間辦公室(供博士後研究員與研究生使用)、小型研討室。

(3)於次震宇宙館無償使用空間之水電費，比照校內教學單位之收費標準收費；借用研究計畫辦公室之水電費，依研究計畫辦公室收費標準計收。

(4)本期執行期間得優先租用次震宇宙館之空間，其租金不高於現行之標準。

六、 其它相關配合措施：請詳細說明所提供之各項設備、學人宿舍、裝修維護費、水電雜支、停車、行政支援...等：

(1)依本校相關規定提供客座學人宿舍給國內、外長期訪問學者。

(2)國內、外學生至中心訪問期間，得依本校相關規定申請本校學生宿舍。

(3)提供一個月(含)以上中心訪客使用本校體育健身設施收費優惠。

(4)提供參加中心活動的成員停車費優惠。

此致

科技部

申請機構首長（簽章）：

學校用印：

中 華 民 國 109 年 6 月 9 日

4 Appendix

Throughout the Appendix, the following abbreviations are used for the Topical Programs:

- A = Number Theory and Representation Theory
- B=Algebraic Geometry
- C=Differential Geometry and Geometric Analysis
- D=Differential Equation and Stochastic Analysis
- E=Scientific Computing
- F=Interdisciplinary Studies

4.1 List of Workshops and Conferences

Workshop Title	Dates	Venues	Organizers	Group
Workshop on Geometric Evolution Equations and Related fields	2021/03/08-2021/03/09	Online, NCTS	Shu-Cheng Chang (NTU) Martin Guest (Waseda) Yoshihiro Ohnita (Osaka City U & OCAMI) Mao-Pei Tsui (NTU)	C
The 21st Taiwan Geometry Symposium	2021/5/1	NTHU	Chiang River (NCKU) Ho Nan-Kuo (NTHU) Tsai Chung-Jun (NTU) Tsui Mao-Pei (NTU)	C
The Third Japan-Taiwan Joint Conference on Differential Geometry	2021/11/01-2021/11/03	NCTS, Online	Shu-Cheng Chang (NTU) Qing-Ming Cheng (Fukuoka Uni.) River Chiang (NCKU) Martin Guest (Waseda Uni.) Nan-Kuo Ho (NTHU) Miyuki Koiso (Kyushu U & IMI) Yng-Ing Lee (NTU) Yoshihiro Ohnita (Osaka City U & OCAMI) Takashi Sakai (Tokyo Metro U) Mao-Pei Tsui (NTU) Sumio Yamada (Gakushuin Uni.)	C
2021 NCTS Young Dynamics Day	2021/2/19	TKU	Jung-Chao Ban (NCCU) Chih-Hung Chang (NUK) Kuo-Chang Chen (NTHU) Cheng-Hsiung Hsu (NCU) Ting-Hui Yang (TKU)	D
The 11th Japan-Taiwan Joint Workshop for Young Scholars in Applied Mathematics	2021/03/08-2021/03/09	Online	Chueh-Hsin Chang (THU) Chiun-Chuan Chen (NTU) Yan-Yu Chen (NTU) Jann-Long Chern (NTNU) Yung-Fu Fang (NCKU) Chang-Hong Wu (NYCU)	D
Third NCTS PDE Symposium	2021/3/26	NUK	Chun-Hsiung Hsia (NTU) Ying-Chieh Lin (NUK) Kung-Chien Wu (NCKU & NCTS) Tsung-Fang Wu (NUK)	D
ReaDiNet 2021: An Online Conference on Recent Topics in Reaction-Diffusion System, Biology, Medicine and Chemistry	2021/10/25-2021/10/29	Online	Thomas Giletti (IECL) Masaharu Nagayama (RIES) Yueyuan Gao (Hokkaido U)	D
Machine learning on solving partial differential equations	2021/9/2	Online	Yng-Ing Lee (NTU) Te-Sheng Lin (NYCU)	DEF
Opportunities and Challenges in Numerical Algebra (I)	2021/1/27	Online	Tsung-Ming Huang (NTNU) Ren-Cang Li (The Uni. of Texas at Arlington) Tiexiang Li (Southeast Uni.) Wen-Wei Lin (NCTU)	E
Opportunities and Challenges in Numerical Algebra (II)	2021/2/3	Online	Tsung-Ming Huang (NTNU) Ren-Cang Li (The Uni. of Texas at Arlington) Tiexiang Li (Southeast Uni.) Wen-Wei Lin (NCTU)	E

Workshop Title	Dates	Venues	Organizers	Group
Opportunities and Challenges in Numerical Algebra (III)	2021/2/17	Online	Tsung-Ming Huang (NTNU) Ren-Cang Li (The Uni. of Texas at Arlington) Tiexiang Li (Southeast Uni.) Wen-Wei Lin (NCTU)	E
Opportunities and Challenges in Numerical Algebra (VI)	2021/2/24	Online	Tsung-Ming Huang (NTNU) Ren-Cang Li (U of Texas at Arlington) Tiexiang Li (Southeast U) Wen-Wei Lin (NCTU)	E
2021 Conference on Advanced Topics and Auto Tuning in High-Performance Scientific Computing	2021/03/19-2021/03/20	NCU	How-Wei Chen (NCU) Feng-Nan Hwang (NCU) Takahiro Katagiri (Nagoya Uni.) Reiji Suda (U of Tokyo) Weichung Wang (NTU) Yungyu Zhuang (NCU)	E
Opportunities and Challenges in Numerical Algebra (VI)	2021/3/20	Online	Tsung-Ming Huang (NTNU) Zhigang Jia (Jiangsu Normal Uni.) Zhongxiao Jia (Tsinghua Uni.) Wen-Wei Lin (NYCU) Michael Ng (Uni. of Hong Kong)	E
Opportunities and Challenges in Numerical Algebra (VI)	2021/3/27	Online	Tsung-Ming Huang (NTNU) Zhigang Jia (Jiangsu Normal Uni.) Zhongxiao Jia (Tsinghua Uni.) Wen-Wei Lin (NYCU) Michael Ng (Uni. of Hong Kong)	E
NCTS Student Workshop on Scientific Computing	2021/4/24	NYCU	Wei-Fan Hu (NCHU) Tsung-Ming Huang (NTNU) Matthew M. Lin (NCKU) Ming-Cheng Shiue (NYCU) Suh-Yuh Yang (NCU)	E
NCTS (Central Taiwan) computational mathematics workshop	2021/4/30	NCHU	Chen Pengwen (NCHU) Hsieh Po-Wen (NCHU)	E
2021 NCTS Online Workshop on Computational Mathematics and Scientific Computing for Young Researchers	2021/09/10-2021/09/11	Online	Wei-Fan Hu (NCU) Tsung-Ming Huang (NTNU) Matthew M. Lin (NCKU) Ming-Cheng Shiue (NYCU) Suh-Yuh Yang (NCU)	E
Meet physicians, mathematicians and statisticians, part 2	2021/3/20	NCTS	Yu-Lun Lo (CGMH & CGU) Hau-tieng Wu (Duke Uni.)	F
2021 NCTS Spring Day	2021/3/29	NCTS	Ynging Lee (NTU & NCTS)	F
NCTS Symposium on Applied Mathematics	2021/6/28	Online	Chu Chia-Chieh Jay (NTHU) Lin Te-Sheng (NYCU) Tsai Je-Chiang (NTHU)	F
Statistics Symposium in Memory of Wen-Chen Chen	2021/07/01-2021/07/02	Online	Chiang Chin-Tsang (NTU) Duan Naihua (Columbia Uni.) Huang Su-Yun (AS) Hwang Chii-Ruey (AS) Hwang Wen-Han (NCHU) Ing Ching-Kang (NTHU)	F
Interfacing Mathematics and Physical Sciences through Machine Learning	2021/9/4	Online, NCTS	Ming-Lun Hsieh (AS) Ying-Jer Kao (NTU) Weichung Wang (NTU)	F

4.2 Seminar Talks

Seminar Title	Dates	Group
NCTS Seminar on Arithmetic Geometry and Representation Theory	4/12, 4/19, 5/28	A
NCTS Seminar on Number Theory	1/6, 4/7, 5/12, 6/2, 8/16, 9/24, 10/21	A
NCTS Seminar on Representation Theory	7/12, 8/2, 8/30	A
Taipei Postdoc Seminar	3/10, 4/7	A
NCTS Seminar in Birational Geometry	9/2, 9/9, 9/30, 10/7	B
NCTS Seminar on Algebraic Geometry	1/29, 2/26, 3/5, 3/12, 3/19, 3/26, 4/9, 4/16, 4/23, 5/7, 5/14, 5/21, 5/28, 6/4, 6/11, 6/18, 6/18, 7/2, 7/9	B
Seminar of Algebraic Geometry in East Asia	1/8, 1/8, 1/22, 1/22, 2/5, 2/19, 2/19, 3/5, 3/5, 3/19, 3/19, 4/2, 4/2, 4/16, 4/16, 4/30, 4/30, 5/14, 5/14, 5/28, 5/28, 6/18, 6/18, 7/2, 7/2, 7/16, 7/16, 7/30, 7/30, 8/13, 8/1, 8/27, 8/27, 9/10, 9/10, 9/24, 9/24	B
Taipei Postdoc Seminar	1/13, 9/29	B
AS-NCTS Seminar on Geometry	1/15, 2/5, 3/19, 4/26, 6/25, 9/24	C
NCTS Differential Geometry Seminar	1/7, 3/18, 3/25, 4/8, 4/15, 4/22, 4/29, 5/13, 5/20, 5/27, 6/3, 8/5, 8/12, 8/26	C
NCTS International Geometric Measure Theory Seminar	1/20, 3/17, 5/19, 7/21, 9/22	C
NCTS Seminar on Geometry and Topology	4/19, 5/3, 5/10	C
NCTS Webinar on Nonlinear Evolutionary Dynamics	9/28, 10/5	D
2021 NCTS Seminar on Dynamical Systems	9/28, 10/1, 10/12, 10/26	D
NCTS Nonlinear PDE and Analysis Seminar	3/3, 3/10, 3/17, 3/24, 3/31, 4/7, 4/14, 4/21, 4/28, 5/12	D
NCTS Quantum Computing Seminar	3/3, 3/10, 3/17, 3/24, 4/7, 4/14, 4/28, 5/5, 5/12	D
NCTS-South Taiwan Seminar on Dynamical Systems	1/7, 1/14	D

Seminar Title	Dates	Group
Nonlinear Phenomena in Evolutionary Partial Differential Equations	1/5, 1/12, 3/2, 3/9, 3/16, 3/23, 3/30, 4/13, 4/20, 4/27, 5/4, 5/11	D
Taipei Postdoc Seminar	3/24, 4/21	D
NCTS Optimization Seminar	4/13, 4/13	F
NCTS Public Lecture	7/22	F
NCTS Seminar on Applied Mathematics	1/12, 2/25, 3/11, 3/25, 4/8, 4/22, 5/6, 5/20	F
NCTS Seminar on Gene Network Analysis	6/18, 7/9, 8/2, 8/4, 9/10	F
NCTS Seminar on Mathematical Biology	3/19, 4/23, 4/30, 5/28, 6/11, 7/30, 10/1	F
Taipei Postdoc Seminar	5/19	F
NCTS Seminar on Reinforcement Learning	3/8	Others

Date	Speaker	Affiliation	Title of Talk
Program A			
2021-10-21	Breuer Florian	University of Newcastle	TBA
2021-09-24	Hsieh Ming-Lun	AS	Modular Construction of Galois Cohomology Classes and L-values
2021-08-30	Pan Shu-Yen	NTHU	Finite Theta Correspondence of Almost Characters
2021-08-16	Wu Ju-Feng	Concordia University	Adjoint p-adic L-functions and the Geometry of Cuspidal Eigenvarieties for General Symplectic Groups
2021-08-02	Hsu Chun-Hsien	Duke University	Harmonic Analysis on Certain Spherical Varieties
2021-07-12	Cheng Yao	TKU	Rankin-Selberg Integrals for SO(2n+1)xGL(r) Attached to New and Old Forms
2021-06-02	Ülkem Özge	Galatasaray University	Moduli Space of Generalized D-elliptic Sheaves and its Uniformization
2021-05-28	Yu Chia-Fu	AS	The Gauss Problem for Central Leaves
2021-05-12	Okumura Yoshiaki	Toyo University	On Congruence of Galois Representations Attached to A-motives
2021-04-19	Yu Chia-Fu	AS	Cohomological Groups of Algebraic Tori (II)
2021-04-12	Yu Chia-Fu	AS	Cohomological Groups of Algebraic Tori (I)
2021-04-07	Peng Jun-Wen	University of Rochester	An Embedding of the Arboreal Galois Group for PCF Maps
2021-04-07	Namoijam Changningphaabi	NTHU	Hyperderivatives of Periods and Quasi-periods of Anderson t-modules
2021-03-10	Chen Shih-Yu	AS	On Deligne's Conjecture for Symmetric Fourth L-functions of Hilbert Modular Forms
2021-01-06	Kaneko Masanobu	Kyushu University	Genus Character L-functions of Quadratic Orders and Class Number Formulas
Program B			
2021-10-07	Lacini Justin	Kansas University	On Pluricanonical Maps of Varieties of General Type
2021-09-30	Cheltsov Ivan	University of Edinburgh	K-stable Fano Threefolds
2021-09-29	Chen Yen-An	NCTS	Boundedness of Canonical Models of Foliated Surfaces

Date	Speaker	Affiliation	Title of Talk
2021-09-24	Okada Takuzo	Saga University	Birational Geometry of Sextic Double Solids with cA Points
2021-09-24	Hashizume Kenta	University of Tokyo	Adjunction and Inversion of Adjunction
2021-09-10	Choe Insong	Konkuk University	Symplectic and Orthogonal Hecke Curves
2021-09-10	Lee Kyoung-Seog	Miami University	Derived Categories and Motives of Moduli Spaces of Vector Bundles on Curves
2021-09-09	Jiang Chen	Fudan University	Explicit Boundedness of Canonical Fano 3-folds: Known Results and Open Problems
2021-09-02	Cascini Paolo	Imperial College London & NCTS	On the Minimal Model for Algebraically Integrable Foliations
2021-08-27	Jia Jia	National University of Singapore	Surjective Endomorphisms of Affine and Projective Surfaces
2021-08-27	Han Jingjun	Johns Hopkins University	Shokurov's Conjecture on Conic Bundles with Canonical Singularities
2021-08-13	Krämer Thomas	Humboldt Universität Berlin	Big Tannaka Groups on Abelian Varieties
2021-08-13	Liu Jihao	University of Utah	Minimal Model Program for Generalized lc Pairs
2021-07-30	Otabe Shusuke	Tokyo Denki University	Universal Triviality of the Chow Group of Zero-cycles and Unramified Logarithmic Hodge-Witt Cohomology
2021-07-30	Tseng Hsian-Hua	Ohio State University	Relative Gromov-Witten Theory without Log Geometry
2021-07-16	Thuong Le Quy	Vietnam National University	The ACVF Theory and Motivic Milnor Fibers
2021-07-16	Jiang Qing Yuan	The University of Edinburgh	On the Derived Categories of Quot Schemes of Locally Free Quotients
2021-07-09	Chen Jheng-Jie	NCU	Rational Curves on K3 Surfaces
2021-07-02	Chang Chi-Kang	NTU	Existence of Sections for RC Fibrations over Curves

Date	Speaker	Affiliation	Title of Talk
2021-07-02	Chen Yifei	Chinese Academy of Sciences	Jordan Property of Automorphism Groups of Surfaces of Positive Characteristic
2021-07-02	Moon Han-Bom	Fordham University, New York	Point Configurations, Phylogenetic Trees, and Dissimilarity Vectors
2021-06-18	Chen Jheng-Jie	NCU	Updated: The Case of Smooth Fano Varieties
2021-06-18	Chen Jheng-Jie	NCU	The Case of Smooth Fano Varieties
2021-06-18	Jiang Zhi	Shanghai Center for Mathematical Sciences	On Syzygies of Homogeneous Varieties
2021-06-18	Zhou Mingshuo	Tianjin University	Moduli Space of Parabolic Bundles over a Curve
2021-06-11	Chang Chi-Kang	NTU	Structure of MRC
2021-06-04	Chen Jungkai	NTU & NCTS	Quotients by Algebraic Relations II
2021-05-28	Pedro dos Santos Joao	Université de Paris	Group Schemes from ODEs Defined over a Discrete Valuation Ring
2021-05-28	Tian Zhiyu	BICMR-Beijing University	Some Conjectures about Kato Homology of Rationally Connected Varieties and KLT Singularities
2021-05-28	Lai Ching-Jui	NCKU	Quotients by Algebraic Relations I
2021-05-21	Chang Chih-Wei	NCTS	Rational Chain Connectedness
2021-05-14	Wen David	NCTS	Unirational, Rationally Connected, (Very) Free Rational Curves
2021-05-14	Shin YongJoo	Chungnam National University	Complex Minimal Surfaces of General Type with p_g= 0 and K^2 = 7 via Bidouble Covers
2021-05-14	Odaka Yuji	Kyoto University	On (various) Geometric Compactifications of Moduli of K3 Surfaces
2021-05-07	Wen David	NCTS	Bend-Break Lemma
2021-04-30	Yasuda Takehiko	Osaka University	On the Isomorphism Problem of Projective Schemes

Date	Speaker	Affiliation	Title of Talk
2021-04-30	Gu Yi	Soochow University	On the Equivariant Automorphism Group of Surface Fibrations
2021-04-23	Brivio Iacopo	NCTS	(Part I - Talk 6) Algebraization of Formal Moduli
2021-04-16	Chen Hsin-Ku	NTU	(Part I - Talk 5) Existence of Miniversal Family and Pro-representability
2021-04-16	Lin Yu-Shen	Boston University	Special Lagrangian Fibrations in Log Calabi-Yau Surfaces and Mirror Symmetry
2021-04-16	Lai Kuan-Wen	University of Massachusetts, Amherst	On the Irrationality of Moduli Spaces of K3 Surfaces
2021-04-09	Nguyen Bin	NCTS	(Part I - Talk 4) Introduction to Formal Moduli (II)
2021-04-02	Česnavičius Kęstutis	Université Paris-Saclay	Grothendieck--Serre in the Quasi--split Unramified Case
2021-04-02	Zheng Weizhe	Morningside Center of Mathematics	Ultraproduct Cohomology and the Decomposition Theorem
2021-03-26	Nguyen Bin	NCTS	(Part I - Talk 4) Introduction to Formal Moduli
2021-03-19	Lee Ting-Yu	NTU	(Part I - Talk 3) Obstruction for Deformations II
2021-03-19	Kemeny Michael	University of Wisconsin–Madison	Universal Secant Bundles and Syzygies
2021-03-19	Li Zhiyuan	Shanghai Center for Mathematical Sciences	Twisted Derived Equivalence for Abelian Surfaces
2021-03-12	Lee Ting-Yu	NTU	(Part I – Talk 3) Obstruction for Deformations
2021-03-05	Chang Chi-Kang	NTU	(Part I – Talk 2) Obstruction for Higher Order Deformations
2021-03-05	Zhong Guolei	National University of Singapore	Fano Threefolds and Fourfolds Admitting Non-isomorphic Endomorphisms
2021-03-05	Xie Junyi	Centre national de la recherche scientifique	Some Boundedness Problems in Cremona Group

Date	Speaker	Affiliation	Title of Talk
2021-02-26	Brivio Iacopo	NCTS	(Part I – Talk 1) First order deformations and construction of T^i functors
2021-02-19	Feyzbakhsh Soheyla	Imperial College London	An Application of a Bogomolov-Gieseker Type Inequality to Counting Invariants
2021-02-19	Won Joonyeong	KIAS	Sasaki-Einstein and Kähler-Einstein Metric on 5-manifolds and Weighted Hypersurfaces
2021-02-05	Zhang Lei	UST of China	Counterexample to Fujita Conjecture in Positive Characteristic
2021-01-29	Odaka Yuji	Kyoto University	K-stability/K-moduli and Related Topics
2021-01-22	Hu Fei	University of Oslo	Some Comparison Problems on Correspondences
2021-01-22	Tanimoto Sho	Kumamoto University	Classifying Sections of Del Pezzo Fibrations
2021-01-13	Hsu You Hung	NCTS	Exceptional Collections, t-structures and Categorical Action
2021-01-08	Matsuzawa Yohsuke	Brown University	Vojta's Conjecture and Arithmetic Dynamics
2021-01-08	Shen Junliang	Massachussetts Institute of Technology	Intersection Cohomology of the Moduli of 1-dimensional Sheaves and the Moduli of Higgs Bundles
Program C			
2021-09-24	Wu Kuang-Ru	AS	Positively Curved Finsler Metrics on Vector Bundles
2021-09-22	Figalli Alessio	ETH Zürich	Free boundary regularity in the Stefan problem
2021-08-26	Lee Tang-Kai	Massachusetts Institute of Technology	Self-shrinkers and Entropy Minimizers for the Mean Curvature Flow
2021-08-12	Chen Po-Ning	University of California, Riverside	Supertranslation Invariance of Angular Momentum
2021-08-05	Chiu Shih-Kai	University of Notre Dame	Calabi-Yau Metrics with Maximal Volume Growth

Date	Speaker	Affiliation	Title of Talk
2021-07-21	Bate David	University of Warwick	A non-linear Besicovitch–Federer projection theorem for metric spaces
2021-06-25	Cheng Da Rong	University of Waterloo	Existence of Constant Mean Curvature 2-spheres
2021-06-03	Wickramasekera Neshan	University of Cambridge	Allen--Cahn Equation and the Existence of Prescribed-mean-curvature Hypersurfaces
2021-05-27	Tonegawa Yoshihiro	Tokyo Institute of Technology	On the Brakke Flows Starting from Minimal Hypersurfaces with Flat Singularities
2021-05-19	Toro Tatiana	University of Washington	Geometric Measure Theory: a powerful tool in Potential Theory
2021-05-13	Kao Lien-Yung	George Washington University	Counting, Equidistribution and Entropy Gap at Infinity
2021-05-10	Liao Hsuan-Yi	NTHU	Formal Exponential Maps for Graded Manifolds
2021-05-03	Liao Hsuan-Yi	NTHU	A CFO Approach to Derived Differential Geometry II
2021-04-29	Imagi Yohsuke	Shanghai Tech University	Floer Theory in Cotangent Bundles
2021-04-26	Lempert László	Purdue University	On the Adjoint Action of Symplectomorphism Groups
2021-04-22	Wu Kuang-Ru	AS	A Dirichlet Problem for Flat Hermitian Metrics
2021-04-19	Chiu Sheng-Fu	AS	From Energy-Time Uncertainty to Symplectic Topological Displacement
2021-04-15	Wood Albert	NCTS	Lagrangian Mean Curvature Flow with Boundary
2021-04-08	Sarquis Aiex Nicolau	NTNU	Index Estimates in Geometry
2021-03-25	Lin Chao-Ming	University of California, Irvine	Deformed Hermitian--Yang--Mills Equation on Compact Hermitian Manifolds and Some Applications
2021-03-19	Hsieh Chun-Chung	AS	Knot Theory, not a Theory
2021-03-18	Madnick Jesse	NCTS	The Jacobi Spectrum of Null-Torsion Holomorphic Curves in the 6-Sphere

Date	Speaker	Affiliation	Title of Talk
2021-03-17	Schulze Felix	University of Warwick	Mean Curvature Flow with Generic Initial Data
2021-02-05	Kao Lien-Yung	George Washington University	Counting, Equidistribution and Entropy Gap at Infinity
2021-01-20	Wei Juncheng	University of British Columbia	Second Order Estimates for Interfaces of Allen-Cahn
2021-01-15	Foo Wei Guo	AS	Equivalence Problem of 5-dimensional Real Hypersurfaces of Type C2,1
2021-01-07	Wood Albert	NCTS	Singularities of Equivariant Lagrangian Mean Curvature Flow
Program D			
2021-10-26	Zhang Zongfan	Sichuan University	The Entropy of Multiplicative Subshifts on Trees
2021-10-12	Chen Changhao	Anhui University	On Measure Theory of Weyl Sums
2021-10-05	de Rijk Björn	Universität Stuttgart	Stability of Pattern-forming Fronts in the Complex Ginzburg-Landau Equation with a Quenching Mechanism
2021-10-01	Peng Jun-Wen	NCTS	Overgroups of the Arboreal Representative of PCF Polynomials
2021-09-28	Fiedler Bernold	Freie Universität Berlin	Calculus of Variations: the “Forgotten” Connecting Orbits
2021-09-28	Wu Yu-Liang	University of Oulu	Topological Entropy on Trees
2021-07-22	Fang Chi-Tai	NTU	Mathematics of Controlling Epidemic
2021-05-12	Chen Yuan-Chieh	NTU	Quantum Algorithms (I)
2021-05-12	Hong Guo-Dong	NTU	The Property of Collision Operator (X)
2021-05-11	Kow Pu-Zhao	NTU	An Introduction to Inverse Problems and Some Recent Progresses
2021-05-05	Chen Yuan-Chieh	NTU	Quantum Circuits (V)
2021-05-04	Chang Chun-Wei	NCTS	Inferring Natural Dynamical Systems Based on Equation-free Empirical Dynamical Modelling
2021-04-28	Su Jhe-Kuan	NTU	The Property of Collision Operator (IX)

Date	Speaker	Affiliation	Title of Talk
2021-04-28	Chen Yuan-Chieh	NTU	Quantum Circuits (IV)
2021-04-27	Chen Shih-Hsin	NTU	On Mathematical Analysis of Synchronization for Forced Kuramoto Oscillators
2021-04-21	Chen Yan-Yu	NTU	Global Dynamics on One-dimensional Excitable Media
2021-04-21	Su Jhe-Kuan	NTU	The Property of Collision Operator (VIII)
2021-04-20	Lee Yi-Yun	NTU	Numerical Propagation Speed of Evolution Solution in Lotka-Volterra Model
2021-04-14	Bae Junsik	NCTS	The Property of Collision Operator (VII)
2021-04-14	Chen Yuan-Chieh	NTU	Quantum Circuits (III)
2021-04-13			Special Event: Ignite Talks
2021-04-07	Bae Junsik	NCTS	The Property of Collision Operator (VI)
2021-04-07	Chen Yuan-Chieh	NTU	Quantum Circuits (II)
2021-03-31	Bae Junsik	NCTS	The Property of Collision Operator (V)
2021-03-30	Shi Zhi-Hao	NTU	Traveling Wave Solution for a Stage Structure Model
2021-03-24	Chen Yuan-Chieh	NTU	Introduction to Quantum Computing
2021-03-24	Chuang Ping-Han	NTU	The Property of Collision Operator (IV)
2021-03-24	Foo Wei Guo	AS	Some Remarks on Second Order Ordinary Differential Equations
2021-03-23	Chen Yuan-Chieh	NTU	Some Results on the Hydrodynamic Limit of Boltzmann Equation
2021-03-17	Su Jhe-Kuan	NTU	The Property of Collision Operator (III)
2021-03-17	Chen Yuan-Chieh	NTU	Introduction to Quantum Computing (III)
2021-03-16	Chuang Ping-Han	NTU	A Revisit of the Velocity Averaging Lemma: On the Regularity of Stationary Boltzmann Equation in a Bounded Convex Domain
2021-03-10	Chen Yuan-Chieh	NTU	Introduction to Quantum Computing (II)
2021-03-10	Hong Guo-Dong	NTU	Properties of the Collision Operator (II)
2021-03-09	Yain Tian-Li	NTU	Some Idea to Identify the ODE System
2021-03-03	Chen Yuan-Chieh	NTU	Introduction to Quantum Computing
2021-03-03	Hong Guo-Dong	NTU	Properties of the Collision Operator (I)
2021-03-02	Dai Jia-Yuan	NCTS	Dynamics of a Morbidostat for Selection of Drug-resistant Mutants

Date	Speaker	Affiliation	Title of Talk
2021-01-14	Chang Chih-Hung	NUK	Relations between Ergodic and Topological Properties (II)
2021-01-12	Hung Chih-Chieh	NCTS	The A2 Sharp Bound for the Hilbert Transform
2021-01-07	Yang Ya-Chu	NUK	Effective Closed Systems on Groups (IV)
2021-01-05	Hsu Yung-Chang	NTU	Time-Frequency Analysis on Modulation Invariant Operators
Program F			
2021-10-01	Wu Yixiang	Middle Tennessee State University	Global Dynamics of a Lotka-Volterra Competition Patch Model
2021-09-10	Bosia Carla	Italian Institute for Genomic Medicine	Practical Examples of MicroRNA-mediated Feed-forward Loops: from Caenorhabditis Elegans to Cancer Cells
2021-08-04	Bosia Carla	Italian Institute for Genomic Medicine	Still on Noise in Genetic Circuits – Introduction to the Role of MicroRNAs in Controlling Gene Expression Noise (II)
2021-08-02	Bosia Carla	Italian Institute for Genomic Medicine	Still on Noise in Genetic Circuits – Biological Hints for Transcriptional Bursts (I)
2021-07-30	Zou Xingfu	University of Western Ontario	Spatial Non-locality Caused by Mobility and Various Types of Delays
2021-07-22	Fang Chi-Tai	National Taiwan University	Mathematics of Controlling Epidemic
2021-07-09	Bosia Carla	Italian Institute for Genomic Medicine	Still on Noise in Genetic Circuits – Biological Hints
2021-06-18	Bosia Carla	Italian Institute for Genomic Medicine	On Genetic Circuits and Noise
2021-06-11	Li Chun-Hsien	National Kaohsiung Normal University	Two-strain Epidemic Models with Vaccinations in Complex Networks
2021-05-28	Browne Cameron	University of Louisiana at Lafayette	Connecting Predator Prey Dynamics in Evolving Pathogen Systems

Date	Speaker	Affiliation	Title of Talk
2021-05-20	Kao Hao-En	NYCU	(Cancelled) Approximation on Matrices of Low Rank
2021-05-19	Yu Guan-Ru	AS	(Cancelled) Bijections for Ranked Tree-Child Networks
2021-05-06	Kao Hao-En	NYCU	Approximation on Matrices of Low Rank
2021-04-30	Vaidya Naveen K.	San Diego State University	COVID-19 Transmission and Control: Mathematical Modeling Perspective
2021-04-23	Chang Chueh-Hsin	Tunghai University	Existence and Stability of the Non-monotone Traveling Wave Solutions of the Three-species Competition-diffusion Systems
2021-04-22	Lin Jun-Jie	NTUHU	Approximation on Matrices of Low Rank
2021-04-13	Lee Yu-Ching	NTHU	Competitive Demand Learning: a Data-Driven Pricing Algorithm
2021-04-13	Ko Chun-Hsu	I-Shou University	Collision-free Trajectory Planning of Robot Walking Helper by Using Optimization
2021-04-08	Lin Jun-Jie	NTHU	Approximation on Matrices of Low Rank
2021-03-25	Kao Hao-En	NYCU	Matrices of Low Rank
2021-03-19	Wang Feng-Bin	Chang Gung University	Applications of Nonlinear Eigenvalue Problems to Ecological Models with Internal Storage
2021-03-11	Kao Hao-En	NYCU	Matrices of Low Rank
2021-02-25	Chang Bo-Wen	NTHU	Matrices of Low Rank
2021-01-12	Chan Michael	CCU	Overexpression of E2F6 suppresses DC Maturation through Epigenetic Silencing of MiR-193a in Ovarian Cancer
Others			
2021-03-08	Ng Kuok Tong	NYCU	Introduction of Reinforcement Learning

4.3 TMS and Other Courses

4.3.1 TMS Courses

Program A	Dates	Organizers
Taiwan Mathematics School: Algebraic Combinatorics II	2021-02-25 / 2021-06-17	Yu Wei-Hsuan (NCU)
Taiwan Mathematics School: Algebraic Number Theory II	2021-02-22 / 2021-06-28	Yu Chia-Fu (AS)
Taiwan Mathematics School: Algebraic Combinatorics I	2021-09-24 / 2022-01-21	Yu Wei-Hsuan (NCU)
Program B		
Taiwan Mathematics School: Arithmetic Dynamics for Rational Dominant Self-maps of Projective Varieties	2021-10-04 / 2021-12-10	Chen Jungkai (NTU)
Taiwan Mathematics School: Introduction to Riemann Surfaces	2021-09-15 / 2022-01-14	Ching-Jui Lai (NCKU)
Program C		
Taiwan Mathematics School: Real Analysis I	2021-09-24 / 2022-01-21	Menne Ulrich (NTNU)
Taiwan Mathematics School: Topics in Geometric Measure Theory II	2021-02-26 / 2021-06-18	Menne Ulrich (NTNU)
Program D		
Taiwan Mathematics School: Multiscale Analysis of Random Processes	2021-02-23 / 2021-06-16	Liu Gi-Ren (NCKU)
Taiwan Mathematics School: Complexity and Symbolic Dynamics of Dynamical Systems	2021-02-22 / 2021-03-22	Ban Jung-Chao (NCCU)
Program E		
Taiwan Mathematics School: Mathematical foundation toward artificial intelligence in medicine	2021-02-23 / 2021-06-17	Wu Hau-Tieng (Duke)
Program F		
Taiwan Mathematics School: Continuation Method: Theory and Application	2021-09-24 / 2022-01-21	Tsai Je-Chiang (NTHU)

4.3.2 Other Courses

Program A	Dates	Organizers
2021 NCTS Summer Course: Polarized Abelian Varieties over Finite Fields	2021-06-09 / 2021-07-07	Yu Chia-Fu (AS)
NCTS Winter Course: Six Aspects of Combinatorial Mathematics	2021-01-18 / 2021-01-22	Yu Wei-Hsuan (NCU), Michael Fuchs (NCCU)
Program D		
NCTS Short Course on Random Walk and Discrete Heat Equation	2021-07-22 / 2021-08-05	Liu Gi-Ren (NCKU)
2021 NCTS Summer Course on Dynamical Systems	2021-07-06 / 2021-07-28	Ban Jung-Chao (NCCU), Chang Chih-Hung (NCCU)
NCTS Short Course on Dynamical Systems	2021-04-16 / 2021-05-07	Ban Jung-Chao (NCCU), Chang Chih-Hung (NCCU)
NCTS Short Course: Introduction to the Basics of Incompressible Navier-Stokes Equation	2021-02-01 / 2021-02-04	Wang Jenn-Nan (NTU)

4.4 List of Visitors

Arrival date	Departure date	Days of visit	English name	Program	Affiliation	Country
2021/1/1	2021/12/31	365	Ryotaro Harada	A	JSPS	Japan
2021/1/1	2021/2/5	36	Chia-Fu Yu	A	Academia Sinica	Taiwan
2021/1/1	2021/12/31	365	Eiichi Bannai	A	Kyushu University	Japan
2021/1/1	2021/12/31	365	Etsuko Bannai	A	Kyushu University	Japan
2021/6/2	2021/9/1	92	Hung Chiang	A	Columbia University	USA
2021/6/11	2021/8/4	55	Chi-Heng Lo	A	Purdue University	USA
2021/1/1	2021/2/21	52	Hsueh-Yung Lin	B	Kavli Institute for The Physics and Mathematics of The Universe	Japan
2021/10/7	2021/12/16	71	Keiji Oguiso	B	University of Tokyo	USA
2021/1/1	2021/8/30	242	Tang-Kai Lee	C	Massachusetts Institute of Technology	USA
2021/1/1	2021/4/30	120	Chao-Ming Lin	C	University of California, Irvine	USA
2021/7/13	2021/9/30	80	Shih-Kai Chiu	C	University of Notre Dame	USA
2021/7/17	2021/7/30	14	Mu-Tao Wang	C	Columbia University	USA
2021/1/1	2021/1/28	28	Jann-Long Chern	D	National Taiwan Normal University	Taiwan
2021/1/1	2021/1/4	4	Jenn-Nan Wang	D	National Taiwan University	Taiwan
2021/1/1	2021/1/31	31	Jin-Cheng Jiang	D	National Tsing Hua University	Taiwan
2021/1/1	2021/6/30	181	Yin-Ting Liao	D	Brown University	USA
2021/1/16	2021/7/7	173	Daniel Eli Spector	D	Okinawa Institute of Science and Technology Graduate University	Japan
2021/3/18	2021/6/23	98	Lien-Yung Kao	D	George Washington University	USA
2021/6/1	2021/8/5	66	Russell Lodge	D	Indiana State University	USA
2021/8/1	2021/12/31	153	Ju-Yi Yen	D	University of Cincinnati	USA
2021/1/1	2021/1/31	31	Weichung Wang	E	National Taiwan University	Taiwan
2021/1/1	2021/6/22	173	Hau-Tieng Wu	F	Duke University	USA
2021/1/11	2021/4/30	110	Hsin-Po Wang	F	University of Illinois at Urbana-Champaign	USA

4.5 Publications

4.5.1 Publications of Key Members

Journal Title	2020 ACK(66)	2020 AFF(48)	2021 ACK(32)	2021 AFF(23)	To Appear(24)
Acta Arithmetica					1
Acta Mathematica Sinica				1	
Adv. Theor. Math. Phys		1			
Advances in Mathematics	1	1	1	1	1
AIMS Mathematics	1	1			
American Naturalist	1	1			
Anal. PDE				1	
Ann. Scuola Norm. Sup. Pisa Cl. Sci		1			
Annales de l'Institut Fourier (Grenoble)				1	1
Annali della Scuola Normale Superiore di Pisa	2				
Annals of Mathematical Sciences and Applications	1				
Applicable Analysis	1		1		
Biomedical Signal Processing and Control	1				
Bull. Inst. Math. Acad. Sin.		1		1	
Calculus of Variations and Partial Differential Equations			1		
Chaos, Solitons and Fractals			1	1	
Computational Statistics and Data Analysis	1				
Computer Methods in Applied Mechanics and Engineering	1				
Contemp. Math. series of AMS					1
Discret Contin Dyn S -A	3	2			
Discrete and Continuous Dynamical Systems - Series B	1		3		2
Diversity				1	
Doc. Math.					
Duke Math. Jour.	1	1		1	1
Ecography			1	1	
Ecology and Evolution			1	1	
Electronic Transactions on Numerical Analysis	1				
Ergodic Theory and Dynamical Systems			1		
European J. Appl. Math.	1	1			
Frontiers in Marine Science			1	1	
Frontiers in Microbiology	1	1			
Global Change Biology	1	1			

Indiana Univ. Math. J.	1	1		1	
International Journal of Mathematics	2	1			
International Mathematical Research Notices	1	1			
Inventiones Mathematicae	1		1		
J. Atmos. Sci.			1		1
J. Geo. Res.					1
J. Math. Fluid Mech.			2		
J. Math. Pures Appl.	1	1			
J. Math. Soc. of Japan		1			
J. Reine Angew. Math.	1				
J. Stat. Mech	1	1			
J. Theor. Nombres Bordeaux		1			
Journal of Algebra	1				
Journal of Algebraic Combinatorics	1	1			
Journal of Clinical Sleep Medicine	1		1		
Journal of Computational Physics	1	1			
Journal of Differential Equations	4	3	3	1	1
Journal of Functional Analysis		1			
Journal of Geometric Analysis			1	1	
Journal of Global Optimization	1				
Journal of Mathematical Analysis and Applications			1		
Journal of Mathematical Biology	1	1	1		1
Journal of Mathematical Physics	2	2	1	1	
Journal of Mathematical Sciences	1				
Journal of Mathematical Study		1			
Journal of Number Theory	1				
Journal of Plankton Research					
Journal of Scientific Computing	1		1		
Journal of Statistical Computation and Simulation			1		
Journal of Statistical Physics	1	1			
Journal of Theoretical Biology		2			
Journal of Topology and Analysis	1				
Limnology and Oceanography	1	1			
Manuscripta Mathematica					1
Mathematische Annalen			1	1	1
Mathematical Biosciences	1				
Mathematical Biosciences and Engineering	1				

Mathematical Proceedings of Cambridge Philosophical Soc.		1			
Mathematical Research Letters	1				
Mathematische Zeitschrift					2
Michigan Mathematical Journal					1
mSystem	1	1			
Nagoya Math Journal				1	1
Nature Communications	1	1			
Nonlinear Anal.	2	2	1		2
Nonlinearity		1	1		1
Pacific Journal of Mathematics	1	1			
Phys. D,	1	1			
PLOS ONE	1		1	1	
Proc. London Math. Soc.	1	1			
Proceedings of RIMS Conference		1			
Progress in Oceanography	1	1			
Q. Appl. Math.					1
Real Analysis Exchange			1	1	1
RIMS K\^oky\^uroku Bessatsu				1	1
Scientific Reports			1	1	
Sensors	1				
SIAM Journal on Imaging Sciences	2				
SIAM Journal on Mathematical Analysis	1	1			
SIAM Journal on Scientific Computing	1				
Statistics in Medicine	1		1		
Studia Math	1				
Taiwanese Journal of Mathematics	1	2			
The Quarterly Journal of Mathematics				1	
Transactions of the American Mathematical Society				1	
Transformation Groups		1			1
WIREs Computational Statistics	1				

4.5.2 Publications of Postdoctoral Fellows

Author	Title	Journal	To Appear	ACK	AFF	Year
Andrea Galasso	Equivariant asymptotics of Szegö kernels under Hamiltonian SU(2)-actions	Asian Journal of Mathematics, Volume 24 (2020), Number 3, Pages: 501 – 532			Y	2020
Bharathwaj Palvannan	Codimension two cycles in Iwasawa theory and tensor product of Hida families.	Math. Ann.	Y		Y	2020
Bin Nguyen	New examples of canonical covers of degree 3	Math. Nachr.	Y	Y	Y	2021
Bin Nguyen	Some infinite sequences of canonical covers of degree 2	Advances in Geometry, Volume 163, Issue 1, pp 143–148 (2021)			Y	2021
Bin Nguyen	Some unlimited families of minimal surfaces of general type with the canonical map of degree 8	Manuscripta Mathematica, Volume 163, Issue 1-2, pp 13–25 (2020)			Y	2020
Changningphaabi Namoijam	Algebraic relations among hyperderivatives of periods and logarithms of Drinfeld modules	arXiv:2103.09485				2021
Changningphaabi Namoijam	Hyperderivatives of periods and quasi-periods for Anderson t-modules	arXiv:2103.05836				2021
Chien Hsun Wang	Stability conditions and braid group actions on affine A type quivers	Journal of algebra and application	Y		Y	2021
Chih-Wei Chang	Asymptotic orders of vanishing along base loci separate Mori chambers	Pacific J. Math. 304(2020), no. 1, 55–64		Y	Y	2020
Chun-Wei Chang	Long-term warming destabilizes aquatic ecosystems through weakening biodiversity-mediated causal networks	Global Change Biology 26(11): 6413-6423			Y	2020
Chun-Wei Chang	Body size, light intensity and nutrient supply determine plankton stoichiometry in mixotrophic plankton food webs	American Naturalist 195(4):E100-E111				2020
David Wen	Higher Dimensional Elliptic Fibrations and Zariski Decomposition	Comm. in Contemporary Mathematics	Y		Y	2020
Jesse Madnick	The Mean Curvature of Special Lagrangian 3-Folds in SU(3)-Structures with Torsion	J. Geom. Phys. 162			Y	2021
Jesse Madnick	The Mean Curvature of First-Order Submanifolds in Exceptional Geometries with Torsion	Ann. Glob. Anal. Geom. 59			Y	2021
Jia-Yuan Dai	Ginzburg-Landau spiral waves in circular and spherical geometries	SIAM Journal on Mathematical Analysis, 53(1), p. 1004–1028		Y	Y	2021
Kazuki Tokimoto	Local Langlands correspondence for regular supercuspidal representations of GL(n)	International Mathematics Research Notices, no. 3, 2007-2073, 5-year IF=1.422, 0 citations				2021
Kazuki Tokimoto	Affinoids in the Lubin-Tate perfectoid space and special cases of the local Langlands correspondence	Mathematische Annalen, 377, 1339-1425, 5-year IF=1.659, 2 citations				2020

Author	Title	Journal	To Appear	ACK	AFF	Year
Oğuz Gezmiş	Trivial multiple zeta values in Tate algebras	Int. Math. Res. Not., Art. ID rnab104, 65 pp				2021
Oğuz Gezmiş	On Drinfeld modular forms of higher rank and quasi-periodic functions	arXiv:2101.11819				2020
Oğuz Gezmiş	Deformation of multiple zeta values and their logarithmic interpretation in positive characteristic	Documenta Mathematica 25, 2355--24211(2020)				2020
Peng-Jie Wong	Refinements of strong multiplicity one for GL(2)	Mathematical Research Letters	Y		Y	2021
Peng-Jie Wong	Cyclicity and exponents of CM elliptic curves modulo p in short intervals	Transactions of the American Mathematical Society 373 (2020) 8725-8749			Y	2020
Peng-Jie Wong	On Selberg's central limit theorem for Dirichlet L-functions	Journal de Théorie des Nombres de Bordeaux 32 (2020) 685-710			Y	2020
Peng-Jie Wong	On the moments of torsion points modulo primes and their applications	International Journal of Number Theory (2020)			Y	2020
Peng-Jie Wong	On generalizations of the Titchmarsh divisor problem	Acta Arithmetica 193 (2020) 321-337			Y	2020
Peng-Jie Wong	Bombieri-Vinogradov theorems for modular forms and applications	Mathematika 66 (2020) 200-229			Y	2020
Sanghyuck Moon	Partly clustering solutions of nonlinear Schrödinger systems with mixed interactions	J. Funct. Anal.	Y		Y	2021
Sanghyuck Moon	Least energy solution for a scalar field equation with a singular nonlinearity	Proc. Roy. Soc. Edinburgh Sect. A, 151 (2021), 93-109			Y	2021
Ser-Wei Fu	Cylinder curves in finite holonomy flat metrics	Journal of Topology and Analysis (2021)			Y	2021
Ser-Wei Fu	Flat grafting deformations of quadratic differentials on surfaces	Geom. Dedicata (2021)			Y	2021
Yasuhiro Terakado	Hecke eigensystems of automorphic forms (mod p) of Hodge type and algebraic modular forms	Mathematische Annalen.				2021
Yasuhiro Terakado	Canonical subgroups of generalized Drinfeld modules	Research in Number Theory, Vol. 7, Article number 8				2021

Author	Title	Journal	To Appear	ACK	AFF	Year
Yi-Sheng Wang	Numerical irreducible criteria for handelbody links	Topology Appl. 284, 107361 (2020)			Y	2020
Yi-Sheng Wang	A complete invariant for connected surfaces in the 3-sphere	J. Knot Theory Ramification, 29, 1, 1950091 (2020)			Y	2020
Yi-Sheng Wang	Topological K-theory with coefficients and the e-invariant	Rocky Mountain J. Math, 50, 1 (2020), 281–318			Y	2020
You-Hung Hsu	Exceptional collections, t-structures, categorical actions-sl2 case,		Y		Y	2021

4.6 Undergraduate Research and Summer Research Program

4.6.1 URP

NCTS URP 2020 (2020.10.1-2021.6.30)

主題名稱	指導老師	學員 1	學員 2	學員 3
1. Denoising the EEG signal using optimal shrinkage to correct biased singular value	吳浩榳 (Duke U.)	邱能泰 (陽明醫)	郭倍誠 (陽明醫)	
2. Tamagawa numbers of CM tori	余家富 (中研院)	梁珮欣 (中山應數)	楊新怡 (台大數學)	
3. Improvement of generalization of Larman-Rogers-Seidel's theorem	俞韋亘 (中央大學)	高暐竣 (台大數學)	葉政叡 (中央數學)	
5. Computation of irreducible characters and branching rules for A_n	彭勇寧 (中央大學)	林明駿 (中央數學)		
6. Finite difference method for PDEs	曾昱豪 (高雄大學)	陳重凱 (高雄應數)	郭昱婕 (高雄應數)	
7. Classification theory on Markov tree shift	班榮超 (政治大學)	古賀琳 (政大應數)		
8. Flux-based Bifurcation Analysis in Reaction Network Systems	蔡志強 (清華大學)	森田展宏 (清大數學)		
9. Vortex Dynamics in Numerical Models	郭鴻基 (台灣大學)	凌文海 (台大大氣)	黃懷逸 (台大大氣)	李琦文 (台大大氣)
10. Mathematical models of COVID-19 with diffusion effects and their data forecast	陳建隆 (師範大學)	陳彥嘉 (師大數學)		
11. Application of Fokas's Method on PDE	黃信元 (交通大學)	許哲瑋 (交大應數)		
12. Topics on Free Probability	黃皓瑋 (中山大學)	盧德倫 (台大數學)	馬宗儀 (台大數學)	李柏駿 (政大應數)

NCTS URP 2021 (2021.10.1-2022.6.30)

主題名稱	指導老師	學員 1	學員 2	學員 3
1. 光學相位重建問題	陳鵬文 (中興大學)	王淑瑄 (中興應數四)		
2. Riemann-Hilbert 方法在可積系統的應用	黃信元 (交通大學)	姜鈞 (中山應數四)		
3. Bispectral analysis and its applications	吳浩榳 (Duke U.) 沈俊嚴 (台灣大學)	顏逸儒 (台大數學四)	張詠信 (台大數學四)	
4. Algebraic number theory towards local-global principles in arithmetic geometry	李庭諭 (台灣大學)	劉耀聰 (台大數學三)	李永丞 (台大數學三)	
5. Topics in Poisson geometry	廖軒毅 (清華大學)	張華炘 (清大數學二)		
6. Data Science with Application to Sleep Stage Automatic Scoring	劉聚仁 (成功大學)	邱能泰 (陽明交大醫學四)	郝嘉誠 (陽明交大電機三)	曾以諾 (成大數學四)
7. On Arithmetic of Multinorm One Tori 多重範一代數環面算術研究	余家富 (中研院)	洪梵雲 (台大數學四)	黃俊皓 (台師大數學三)	
8. On K3 surfaces: moduli spaces, automorphism groups and fibrations	陳榮凱 (台大數學)	陳沅蓁 (台大數學三)	蘇品丞 (台大物理五)	陳毅鴻 (台大數學四)
9. Combinatorial aspects of non-commutative probability theory and its application in data science	黃皓瑋 (中山大學)	蕭明 (台大數學二)	周子涵 (政大應數三)	

4.6.2 USRP

NCTS USRP 2021

主題名稱	指導老師	助教	學員 1	學員 2	學員 3	學員 4
1.代數環面計算問題之探討 Computational problems of certain algebraic tori	余家富 (中研院) 李庭諭 (台大數學)	梁珮欣	台大數學二 李永丞	台大數學二 張志煥	師大數學二 黃俊皓	中正數學三 陳家湘
2. Combinatorics of finitely many points on the sphere	Eiichi Bannai (九州大學)	俞韋亘	陽明醫學四 邱能泰	中央數學四 葉政叡	台大數學三 林楷庭	中央數學三 易瑋泓
3. 以幾何分析的方法了解近代幾何問題	陳志偉 (中山應數) 李國瑋 (彰師數學)	蔡約秩	台大物理四 (雙修數學) 蘇品丞	高雄大應數四 許智祐	成大數學四 張恒宇	成大數學四 蔣岳霖
4.自由機率論與其應用 Free Probability Theory and its Applications	黃皓瑋 (中山應數)	盧德倫 馬宗儀	台大數學二 蕭明	陽明醫學五 郭倍誠	政大應數三 周子涵	
5.可積系統專題 Topics in Integrable System	黃信元 (交大應數)	王德鈞	中山應數三 姜鈞	陽明交大百川學士一 曹瑋		
6.高維度非自治系統之動態行為 On the dynamics of the multidimensional nonautonomous dynamical systems	班榮超 (政大應數) 張志鴻 (高雄大應數)	蔡承育	政大應數三 邱祥慈	台大機械三 游承澔	政大應數三 古賀琳	
7.深度學習在科學計算上的應用 Applications of Deep Learning for Scientific Computing	胡偉帆 (中央數學)	林靖旻	台大大氣一 許佑杰	政大應數三 胡傳宇	師大數學四 陳樞元	
8.光學相位重建問題	陳鵬文 (中興應數)	林瑞琳	中興應數三 王潞瑄	清大經濟三 (雙修數學) 蔡智予		
9. Study on data analysis and dimensionality reduction	朱家杰 (清大數學) 蘇承芳 (交大應數)	林俊傑	陽明交大電機二 郝嘉誠	清大數學三 (電機輔) 蘇昭憲	清大理學院四 吳善棨	