Representation Theory of Lie Superalgebras and Related Topics

Taipei workshop | 2017

July 3 (Mon.) – July 7 (Fri.)

Rm 202, NCTS
(Astro-Math Bldg., NTU)

NCTS
National Center for Theoretical Sciences
Mathematics in Taiwan

ACADEMIA SINICA

2017 TAIPEI WORKSHOP ON REPRESENTATION THEORY OF LIE SUPERALGEBRAS AND RELATED TOPICS

JULY 3 - 7, 2017

National Center for Theoretical Sciences, TAIPEI, TAIWAN

	July 3 (Mon.)	July 4 (Tue.)	July 5 (Wed.)	July 6 (Thur.)	July 7 (Fri.)
	9:00-9:25 Registration 9:25 OPENING				
Chair	Shun-Jen Cheng	Weiqiang Wang	Jun Hu	Chih-Whi Chen	Chun-Ju Lai
09:30 - 10:30	Weiqiang Wang	Shunsuke Tsuchioka	Chun-Ju Lai	Yiqiang Li	Myungho Kim
10:30 – 11:00	Tea Break	Tea Break	Tea Break	Tea Break	Tea Break
11:00 – 12:00	Yucai Su	Ivan Chi-Ho Ip	Li Luo	Inna Entova-Aizenbud	Euiyong Park
		12:05 Group Photo			
12:00 – 13:30	Lunch				
Chair	Jonathan Kujawa	Yung-Ning Peng	Social Event (Trails in Beitou Area)	Jae-Hoon Kwon	
13:30 – 14:30	Jun Hu	Jae-Hoon Kwon		Jonathan Kujawa	
14:30 – 15:00	Tea Break	Tea Break		Tea Break	
15:00 – 16:00	Simon Goodwin	Satoshi Naito		Kevin Coulembier	
16:00 – 16:20	Tea Break	Tea Break		Tea Break	
16:20 – 17:20	Sean Ian Clark	Se-Jin Oh		Nicholas Davidson	
17:40		Reception 6th floor Lounge Institute of Math			

2017.6.27 updated

Representation Theory of Lie Superalgebras and Related Topics

Canonical Bases for Quantized General Linear and Orthosymplectic Lie Superalgebras

Sean Ian Clark (Northeastern University)

Abstract:
The essential ingredients to Lustzig's original construction of canonical bases for quantized enveloping algebras were the integral form; PBW bases arising from braid isomorphisms; and rank 2 computations. In this talk, I will discuss how essentially the same ideas, with some modifcations, can be used to construct canonical bases for basic type Lie superalgebras, in the case of the half-quantum enveloping superalgebras associated to positive root systems whose simple roots satisfy $(\alpha_i, \alpha_j) \leq 0$ for $i \neq j$. The main ideas will be illustrated with examples in types $A(m|1)$, $A(m|2)$, and $\mathrm{D}(m|1)$. Time permitting, I will discuss obstructions to generalizing this approach.

Homological properties of category O

Kevin Coulembier (University of Sydney)

Abstract:
I will discuss some homological aspects of the theory of the *BGG* category O for classical Lie superalgebras. I will comment on some work on projective dimensions, complexity and classification of blocks done over the last years by several authors. Then I will focus on a new point of view in terms of Frobenius extensions and Gorenstein homological algebra. This allows to reinterpret and extend several known results.

The ABCs of supercategory $\mathcal{O}$

Nicholas Davidson (University of Oklahoma)

Abstract:
In this talk, I will discuss the representation theory of the Lie superalgebra $\mathfrak{q}_n(\mathbb{C})$ in its BGG category $\mathcal{O}$. Work by Chen has reduced many questions about the study of this category to the study of the so-called type A, B, and C blocks. I will recall several conjectures (due to Cheng, Kwon, and Wang) about the structure of the type A and C blocks, and discuss how the techniques of categorical representation theory have been used to prove these conjectures. This talk is based on joint work with Jonathan Brundan.

Representation Theory of Lie Superalgebras and Related Topics

On Finite-dimensional Representations of the Lie superalgebra P(n)

Inna Entova-Aizenbud (Ben Gurion University)

Abstract:
Given a supervector space $V = C^{(n|n)}$ with an odd symmetric bilinear form, the periplectic Lie superalgebra $\mathfrak{p}(n)$ consists of linear transformations preserving this form. This originally appeared in the classification of classical-type Lie superalgebras due to V. Kac: for $n > 2$, this algebra has a simple ideal of codimension 1, which is one of the two "strange" series of simple superalgebras.
In this talk, I will present some of the results we obtained concerning the category $Rep(\mathfrak{p}(n))$ of finite-dimensional representations. Unlike the rest of the classical-type Lie superalgebras, the structure of this category has not been well understood until now.
I will explain the classification of blocks in this category, the combinatorics behind the Kazhdan-Lusztig coefficients, and present a categorical action of the infinite Temperley-Lieb algebra through translation functors.
This is part of a joint project with M. Balagovic, Z. Daugherty, M. Gorelik, I. Halacheva, J. Hennig , M. Seong Im, G. Letzter, E. Norton, V. Serganova, C. Stroppel.

Representations of $\mathfrak{gl}(m|n)$ and its Principal W-algebra

Simon Goodwin (University of Birmingham)

Abstract:
We'll give an overview of joint work with Brundan on the representation theory of the principal W-algebra of $\mathfrak{gl}(m|n)$ and explain its relationship with category $\mathcal{O}$ for $\mathfrak{gl}(m|n)$ via the Whittaker coinvariants functor. The results we'll cover include: explicit computation of Cartan matrices associated to the principal W-algebra; and some applications to the classification of blocks in category $\mathcal{O}$ up to Morita/derived equivalence.

Symmetric Structure for the Endomorphism Algebra of Projective-injective Module in Parabolic Category

Jun Hu (Beijing Institute of Technology)

Abstract:
We show that for any singular dominant integral weight λ of a complex semisimple Lie algebra $\mathfrak{g}$, the endomorphism algebra $B_\lambda^\mathfrak{p}$ of any projective-injective module in the parabolic BGG category $\mathcal{O}_\lambda^\mathfrak{p}$ is a symmetric algebra (as conjectured by Khovanov) extending the results of Mazorchuk and Stroppel for the regular dominant integral weight. Moreover, the endomorphism algebra of any projective-injective module in $\mathcal{O}_\lambda^\mathfrak{p}$ equips with a homogeneous non-degenerate symmetrizing form. This is a joint work with Ngau Lam.

Representation Theory of Lie Superalgebras and Related Topics

Generalized Teichmüller Spaces, Spin Structures, and Ptolemy Transformations

Ivan Chi-Ho, Ip (Kyoto University)

Abstract:
Teichmüller space is a fundamental space that is important in many areas of mathematics and physics. In recent times generalizations of this space have been intensely studied. Examples of such higher Teichmller spaces are given by the so-called super-Teichmller spaces, with structural groups given by supergroups. These appear as a natural object when studying a combinatorial approach to spin structures on Riemann surfaces and the generalization to supermanifolds. In this talk, we give a substantial simplification of the formulation of the spin structures and describe the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ super-Teichmüller space using the analogue of Penner coordinates. (Joint work with Robert Penner and Anton Zeitlin.)

Centers of Semisimple Walled Brauer Algebras

Myungho Kim (Kyung Hee University)

Abstract:
In this talk, a family of commuting elements of the walled Brauer algebra, called the Jucys-Murphy elements, will be introduced. As similar in the case of symmetric groups, the supersymmetric polynomials in the Jucys-Murphy elements belong to the center of the walled Brauer algebra. We show that if the walled Brauer algebra is semisimple, then these supersymmetric polynomials generate the center. We have an analogue of the Jucys-Murphy elements for semisimple quantized walled Brauer algebras and a similar connection between the supersymmetric polynomials in the Jucys-Murphy elements and the center. This is a joint work with Ji Hye Jung.

Combinatorial Categories of Type Q

Jonathan Kujawa (University of Oklahoma)

Abstract:
Recently there is a growing interest in combinatorial categories. That is, categories whose objects, morphisms, composition, etc. are given by, for example, diagrammatic rules. They naturally appear in higher representation theory as well as in new approaches to classical problems. In this talk I will explain several combinatorial categories which arise from the representation theory of Lie superalgebras of type Q. This work is joint with Jonny Comes and Gordon Brown.

Representation Theory of Lie Superalgebras and Related Topics

A Crystal Embedding of Kashiwara-Nakashima Tableaux into Lusztig Data

Jae-Hoon Kwon (Seoul National University)

Abstract:
In this talk, we present an explicit combinatorial algorithm for computing the Lusztig data of Kashiwara-Nakashima tableaux in types B and C associated to a family of reduced expressions of the longest element. Our description of the crystal embedding of KN tabeleaux into the crystal of Lusztig data is simple and elementary in the sense that we use only the Schützenberger's jeu de taquin and RSK algorithm, the combinatorics of type A. We use a spinor model for classical crystals as an intermediate object connecting KN tableaux and Lusztig data, which plays an important role. If time permits, we discuss a superization of this combinatorics and possible connection with representations of quantum superalgebras.

RTT Formalism, Schur Algebras and Quantum Supergroups

Chun-Ju Lai (Max-Planck Institute for Mathematics in Bonn)

Abstract:
The quantum (super)groups are known to admit various realizations for their nature of connecting many areas of mathematics. The original realization of the quantum groups due to Drinfeld-Jimbo is given in terms of the Chevalley generators and the Serre relations; while the tenrary (or RTT) presentation introduced by Faddeev-Reshetikhin-Takhtajan has relations in the form of a matrix equation $RTT = TTR$. There is also a realization in terms of Schur algebras introduced by Beilinson-Lusztig-MacPherson via geometry of partial flags varieties or combinatorics of Hecke algebras. In this talk I will focus on the quantum (super)groups of type A and give a RTT presentation of quantum Schur superalgebras.

σ-Quiver Varieties

Yiqiang Li (University at Buffalo)

Abstract:
I'll present a new class of quiver varieties and its connection with classical geometry and representation theory of symmetric pairs.

An Algebraic Approach to Quantum Schur Duality of Type D

Li Luo (East China Normal University)

Abstract:
Fan and Li introduced two new quantum algebras arising form N-step isotropic flag varieties of type **D**, which admits a Schur-like duality together with the Iwahori-Hecke algebra of type **D**. In this talk, I will give a purely algebraic realization for these quantum algebras and their canonical bases.

Representation Theory of Lie Superalgebras and Related Topics

Standard Monomial Theory for Semi-infinite LS Paths and Semi-infinite Flag Manifolds

Satoshi Naito (Tokyo Institute of Technology)

Abstract:
In this talk, we first explain a combinatorial version of standard monomial theory (SMT) for semi-infinite Lakshmibai-Seshadri (LS) paths.
Then, we give an application of this SMT to the geometric study of semi-infinite flag manifolds. In fact, we give a Pieri-Chevalley type formula for a version of equivariant (with respect to an Iwahori subgroup) K-theory of semi-infinite flag manifolds; this formula describes, in terms of semi-infinite LS paths, the product (in the K-theory) of the structure sheaf of an arbitrary semi-infinite Schubert variety with a line bundle (associated to a dominant weight) on a semi-infinite flag manifold. In order to prove this formula, we provide fundamental results on the formal power series model of a semi-infinite flag manifold including its normality, higher cohomology vanishing, and also a Borel-Weil type theorem.

Some Remark on Multiplicities

Se-jin Oh (Ewha Womans University)

Abstract:
The notion of "multiplicity" is frequently used in mathematics. In this talk, I would like to focus on weight multiplicity and composition multiplicity. In the joint work with Jangsoo Kim and Kyu-Hwan Lee, we observed that, sometimes composition multiplicities appear as weight multiplicities. Based on such observations, we can provide *practically computable closed formulas* for multiplicities of certain infinite families of weight spaces. To do that, we introduced new family of tableaux, named as *rigid tableaux*, and employed the crystal basis theory initiated by Kashiwara.

Specht Modules for Quiver Hecke Algebras of Type C

Euiyong Park (University of Seoul)

Abstract:
In this talk, I will explain about Specht modules for quiver Hecke algebras in type $C_\ell^{(1)}$ and C_∞. We construct and investigate Specht modules S^λ for cyclotomic quiver Hecke algebras in affine type C, which are labelled by multipartitions λ. It is shown that in type C_∞, the Specht module S^λ has a homogeneous basis indexed by standard tableaux of shape λ, which yields a graded character formula and good properties with the exact functors E_i^Λ and F_i^Λ. This talk is based on arXiv:1703.06425, which is a joint work with S. Ariki(Osaka University) and L. Speyer(Osaka University).

Representation Theory of Lie Superalgebras and Related Topics

Highest Weight Vectors of Mixed Tensor Products of General Linear Lie Superalgebras

Yucai Su (Tongji University)

Abstract:
A notion of cyclotomic (or level k) walled Brauer algebras $\mathcal{B}_{k,r,t}$ is present for arbitrary positive integer k. It is proven that $\mathcal{B}_{k,r,t}$ is free over a commutative ring with rank $k^{r+t}(r+t)!$ if and only if it is admissible in some sense. Using super Schur-Weyl duality between general linear Lie superalgebras $\mathfrak{gl}_{m|n}$ and $\mathcal{B}_{2,r,t}$, we give a classification of highest weight vectors of $\mathfrak{gl}_{m|n}$-modules M_{pq}^{rt}, the tensor products of Kac-modules with mixed tensor products of the natural module and its dual. This enables us to establish an explicit relationship between $\mathfrak{gl}_{m|n}$-Kac-modules and right cell (or standard) $\mathcal{B}_{2,r,t}$-modules over $\mathbb{C}$. Further, we find an explicit relationship between indecomposable tilting $\mathfrak{gl}_{m|n}$-modules appearing in M_{pq}^{rt}, and principal indecomposable right $\mathcal{B}_{2,r,t}$-modules via the notion of Kleshchev bipartitions. As an application, decomposition numbers of $\mathcal{B}_{2,r,t}$ arising from super Schur-Weyl duality are determined. This is a joint work with Hebing Rui.

New Parametrization of Irreducible Modular Spin Representations of The Symmetric Group

Shunsuke Tsuchioka (University of Tokyo)

Abstract:
We propose a generalization of Schur regular partitions for each odd integer $p \geq 3$. Applying Kashiwara crystal theory, we prove that the number of partitions of n with this condition is equinumerous to the number of odd p-class regular partitions of n. At $p = 3$, it is Schur's 1926 partition theorem found as a mod 6 analog of the Rogers-Ramanujan partition theorem (RRPT). The statement for $p = 5$ was conjectured by Andrews in 1970s in a course of his 3 parameter generalization of RRPT and proved in 1994 by Andrews-Bessenrodt-Olsson with an aid of computer.
We expect that our parametrization of irreducible modular spin representations of the symmetric groups is the one whose existence was expected before Lascoux-Leclerc-Thibon-Ariki theory. This is a joint work with Masaki Watanabe.

Character Formulae in Category O of Exceptional Lie Superalgebras

Weiqiang Wang (University of Virginia)

Abstract:
Let g be the exceptional Lie superalgebra $D(2|1;a)$ or $G(3)$. We shall describe the character formulae for representations in the BGG category O. This is joint work (work in progress for $G(3)$) with Shun-Jen Cheng.

Organizers:

Chih-Whi Chen (NCTS)

Shun-Jen Cheng (AS)

Yung-Ning Peng (NCU)

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