Workshop on

Inverse Problems and Related Subjects

Time: 5/27 (Sat.)

Venue: Rm 440, NCTS (Astro-Math Bldg., NTU)

National Center for
Theoretical Sciences
Mathematics Division, Taiwan

NCTS

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09:00–09:50	Paul Sacks (Iowa State University)
	Transmission inverse problem with reduced information about the source
09:50–10:20	**Break**
10:20–10:50	Wei-Chuan Wang (NQU)
	Some inverse Problem Related to Reaction-diffusion Models
10:50–11:20	Catalin Carstea (NCTS)
	Size Estimates of Inclusions for the 3D Complex Conductivity Equation from Three Boundary Measurements
11:20–11:30	**Break**
11:30–12:00	Ching-Lung Lin (NCKU)
	Doubling Inequalities for the Lame System with Rough Coefficient3
12:00–14:00	**Lunch Break**
14:00–14:50	Chun-Kong Law (NSYSU)
	Ambarzumyan Problems for Quantum Graphs
14:50–15:20	**Break**
15:20–15:50	Kuo-Ming Lee (NCKU)
	Transmission Scattering Problem via a DtN Map
15:50–16:20	Lunghui Chen (CCU)
	On the Near Field Basis in Radially Symmetric Interior Transmission Problem and the Well-Posedness
16:20–16:30	**Break**
16:30–17:00	Rulin Kuan (NCKU)
	Strong Unique Continuation for Two-dimensional Elliptic Systems with Gevrey Coefficients
17:00–17:30	Manas Kar (NCTS)
	Size Estimates for the p-Laplace Equation with One Measurement in Plane
18:00–	**Dinner Reception**

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Transmission Inverse Problem with Reduced Information about the Source

Paul Sacks (Iowa State University)

Abstract:
In commonly studied coefficient identification problems in wave propagation it is assumed that the wavefield data being measured arises in response to a known source function, often an idealization such as a delta function. However it may be the case that the source is itself difficult or impossible to know with any precision, thus some works have treated the source itself, subject to some constraints, as an unknown in the problem, together with the coefficient(s) of interest. In this talk we will discuss two problems of this type involving the determination of an unknown impedance in a one dimensional wave equation from corresponding transmission data. In the first version it is assumed that the phase of the Fourier transform of the source function is known, but not its amplitude. In the second version no specific knowledge of the source is assumed, but some restrictions are imposed on the locations of the complex zeros of its Fourier transform. The strategy in both cases is to first infer the data for a related inverse spectral problem.

Ambarzumyan Problems for Quantum Graphs

Chun-Kong Law (NSYSU)

Abstract:
We shall give a short survey on the development of Ambarzumyan problems on the interval as well as on quantum graphs. One key result is the Ambarzumyan-Carlson-Pivovarchik Theorem which says that if a quantum tree has edgelengths with rational ratios with the total length L, and has eigenvalues of the form $(\frac{km_0\pi}{L})^2$, then the potential functions $q_i = 0$. We shall discuss several extensions of this theorem.

Doubling Inequalities for the Lame System with Rough Coefficient3

Ching-Lung Lin (NCKU)

Abstract:
In this paper we study the local behavior of a solution to the Lame system when the Lame coefficients λ and μ satisfy that μ is Lipschitz and λ is essentially bounded in dimension $n > 1$. One of the main results is the local doubling inequality for the solution of the Lame system. This is a quantitative estimate of the strong unique continuation property. Our proof relies on Carleman estimates with carefully chosen weights. Furthermore, we also prove the global doubling inequality, which is useful in some inverse problems.

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Transmission Scattering Problem via a DtN Map

Kuo-Ming Lee (NCKU)

Abstract: For a plane incident wave $u^i(x,d) := \mathrm{e}^{\mathrm{i}k<x,d>}$ with a unit vector d indicating the direction of propagation and a wave number $k > 0$, we consider the following scattering problem

Problem 1. (Transmission scattering problem)
Let $D \subset \mathbb{R}^2$ be a simply connected domain with C^2 boundary ∂D, find two functions $u_1 \in C^2(\mathbb{R}^2 \setminus \bar{D}) \cap C(\mathbb{R}^2 \setminus D)$ and $u_2 \in C^2(D) \cap C(\bar{D})$ satisfy

1. $$\begin{cases} \Delta u_1 + k_1^2 u_1 = 0, & \text{in } \mathbb{R}^2 \setminus \bar{D} \\ \Delta u_2 + k_2^2 u_2 = 0, & \text{in } D \end{cases} \tag{1}$$

   with $k_1, k_2 > 0$.

2. The transmission boundary conditions

   $$\begin{cases} u_1 = u_2 & \text{on } \partial D \\ \frac{\partial u_1}{\partial \nu} = \frac{\partial u_2}{\partial \nu} & \text{on } \partial D \end{cases} \tag{2}$$

   where the normal derivatives

   $$\frac{\partial u_i}{\partial \nu}(x) := \lim_{h \to 0^+} \left\langle \nu(x), \operatorname{grad} u(x - (-1)^i h\nu(x)) \right\rangle, \quad x \in \partial D \tag{3}$$

   exist uniformly on ∂D.

3. The Sommerfeld radiation condition

   $$\lim_{r\to\infty} \sqrt{r} \left(\frac{\partial u_1}{\partial \nu} - iku_1 \right) = 0, \qquad r := |x| \tag{4}$$

   uniformly for all directions $\hat{x} := \frac{x}{|x|}$

and the corresponding inverse problem

Problem 2. (Inverse transmission scattering problem)
Find the shape and location of the penetrable scatterer from the knowledge of the measured far-field pattern.

by proposing a DtN map.

Strong Unique Continuation for Two-dimensional Elliptic Systems with Gevrey Coefficients

Rulin Kuan (NCKU)

Abstract:
In this talk, we study the strong unique continuation property (SUCP) for the general elliptic systems of two variables. We assume all the coefficients belong to Gevrey class and the system has simple characteristics. We reduce and transform the original systems to a diagonal but larger second order elliptic systems and prove SUCP by establishing appropriate Carleman estimates for this reduced system.

On the Near Field Basis in Radially Symmetric Interior Transmission Problem and the Well-Posedness

Lunghui Chen (CCU)

Abstract:

In this talk, we will address on the interior transmission problem of the Helmholtz equation. The spectra of interior transmission problem is the zero set of certain entire functional determinant. In particular, the eigenvalues are the zeros of the class of sine-type function. Whenever provided a sufficient quantity of uniformly distributed zeros, there exists a series of exponential polynomials which completes a Riesz basis in a suitable ball. As an application, we will discuss the inverse well-posedness of the index of refraction when certain information on the eigenvalue distribution is provided.

The interior transmission eigenvalues play a role in the numerical computation and theoretical analysis in the inverse scattering theory. We deduce the Riesz basis to provide a numerical scheme to analyze the convergence and the stability of approximating series such as those in the finite element method, optimization methods, and Fourier expansion method.

Some Inverse Problem Related to Reaction-diffusion Models

Wei-Chuan Wang (NQU)

Abstract: A basic reaction-diffusion model

$$U_t - U_{xx} = \lambda U - w(x)|U|^p U$$

is considered. It is known that steady states of the above problem possess a finite number of nodes. Owing to such oscillation properties and under some certain assumption, we propose and solve an inverse nodal problem for the function $w(x)$. We also intend to extend this issue to a FitzHugh-Nagumo type elliptic system. This is a joint work with Y.S. Choi and C.K. Law.

Size Estimates of Inclusions for the 3D Complex Conductivity Equation from Three Boundary Measurements

Catalin Carstea (NCTS)

Abstract: In this talk I will consider a the conductivity equation with complex conductivity for a three dimensional object which consists of two homogeneous phases. Using a variational method (the "translation method") I will show how one may obtain lower and upper bounds for the relative volume of one of the phases from just three boundary measurements.

Size Estimates for the p-Laplace Equation with One Measurement in Plane

Manas Kar (NCTS)

Abstract: In this talk, we will discuss the problem of estimating the size of an inclusion embedded in an object laying in the two dimensional domain. We consider the p-Laplace equation as a model problem in this case. Using only one voltage-current measurement, we give upper and lower bounds of the size of the inclusion. This is a joint work with Jenn-Nan Wang.

Organizer: Jenn-Nan Wang (NTU)

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