# ON THE NUMBER OF COMMUTING MATRICES AND KAC POLYNOMIALS

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Abstract. There is a classical result of Feit and Fine on the numbers of commuting matrices with coefficients in a finite field $\mathbb{F}_q$ and recently it was used by Behrend, Bryan and Szendröi to compute the degree zero (motivic) Donaldson-Thomas invariants of $\mathbb{C}^3$. In this talk, we consider an extension of their theorem to the context of representations of quivers in which the Kac polynomials will appear and we will see how the positivity conjecture of the DT invariants are linked to Kac's positivity conjecture. Throughout the content, we will introduce the (category of) representations of quivers and the associated Kac polynomials and refined DT invariants and compute them in some simple examples. Then we will discuss their basic properties, including integrality, positivity, wall-crossing phenomena, their relations with Kac-Moody algebras and sketch the "proof" of Kac conjecture.