A SMALL-GAIN THEOREM FOR NONLINEAR STOCHASTIC SYSTEMS WITH INPUTS AND OUTPUTS I: ADDITIVE WHITE NOISE

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Abstract. This paper studies a small-gain theorem for nonlinear stochastic equations driven by additive white noise in both trajectories and stationary distribution. Motivated by the most recent work of Marcondes de Freitas and Sontag [*SIAM J. Control Optim.*, 53 (2015), pp. 2657-2695],, we firstly define the *"input-to-state characteristic operator"* $\mathcal{K}(u)$ of the system in a suitably chosen input space via backward Itô integral, and then for a given output function h, define the "*gain operator*" as the composition of output function h and the input-to-state characteristic operator $\mathcal{K}(u)$ on the input space. Suppose that the output function is either order-preserving or anti-order-preserving in the usual vector order and the global Lipschitz constant of the output function is less than the absolute of the negative principal eigenvalue of linear matrix. Then we prove the so-called *"small-gain theorem"*: the gain operator has a unique fixed point, the image for input-to-state characteristic operator at the fixed point is a globally attracting stochastic equilibrium for the random dynamical system generated by the stochastic system. Under the same assumption for the relation between the Lipschitz constant of the output function and maximal real part of stable linear matrix, we prove that the stochastic system has a unique stationary distribution, which is regarded as a stationary distribution version of small-gain theorem. These results can be applied to stochastic cooperative, competitive and predator-prey systems, or even others.

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