The Decomposition Formula and Stationary Measures for Stochastic Lotka-Volterra Systems with Applications to Turbulent Convection

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Abstract

Motivated by the work of Busse et al. on turbulent convection in a rotating layer, we exploit the long-run behavior for stochastic Lotka-Volterra (LV) systems both in pull-back trajectory and in stationary measure. It is proved stochastic decomposition formula describing the relation between solutions of stochastic and deterministic LV systems and stochastic Logistic equation. By virtue of this formula, it is verified that every pull-back omega limit set is an omega limit set for deterministic LV systems multiplied by the random equilibrium of the stochastic Logistic equation. This formula is used to derive the existence of a stationary measure, its support and ergodicity. We prove the tightness for the set of stationary measures and the invariance for their weak limits as the noise intensity vanishes, whose supports are contained in the Birkhoff center.

The developed theory is successfully utilized to completely classify three dimensional competitive stochastic LV systems into 37 classes. Time average probability measures weakly converges to an ergodic stationary measure on the attracting domain of an omega limit set in all classes except class 27 c). Among them there are two classes possessing a continuum of random closed orbits and ergodic stationary measures supported in cone surfaces, which weakly converge to the Haar measures of periodic orbits as the noise intensity vanishes. In the exceptional class, almost every pull-back trajectory cyclically oscillates around the boundary of the stochastic carrying simplex characterized by three unstable stationary solutions. The limit for the time average probability measures is neither unique nor ergodic. These are subject to turbulent characteristics.

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