# 中央研究院數學研究所

## Institute of Mathematics, Academia Sinica

## Sinica-NCTS
## Geometry Seminar

Speaker： 神島芳宣教授 Professor Yoshinobu Kamishima

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Title：**HOMOGENEOUS SASAKI MANIFOLD** $G/H$ **OF UNIMODULAR LIE GROUP** $G$

Abstract A pseudo-Hermitian structure on a smooth manifold $M$ consists of a pair $(\omega, J)$ where $\omega$ is a contact form, together with a complex structure $J$ defined on a contact subbundle $\ker \omega$. When the Reeb field $A$ for $\omega$ generates a 1-parameter group $T$ of pseudo-Hermitian transformations of $M$, i.e. holomorphic transformations on $\ker \omega$, we call $(M, \omega, J, A)$ a *standard pseudo-Hermitian manifold*, equivalently stated as a *Sasaki manifold*. In addition if $T$ acts properly and freely, then $M$ is said to be a *regular* Sasaki manifold. Suppose that $(G/H, \omega, J)$ is a *simply connected homogeneous Sasaki* manifold. Then it follows that $G/H$ is regular such that $T = S^1$ or $\mathbb{R}$.

**Theorem.** *Let $G$ be a simply connected unimodular Lie group and $H$ a connected compact subgroup. Then a homogeneous Sasaki manifold $G/H$ fibers over a homogeneous Kähler manifold $PG/PH$ of reductive Lie group $PG$ with fiber $T$:*

$$T \longrightarrow G/H \xrightarrow{\ P\ } PG/PH.$$

Moreover, we shall apply this theorem to determine the *unimodular Sasaki groups*. Essentially the above theorem gives a final part of a classification to *homogeneous Vaisman manifolds*. In fact, *locally conformal Kähler* structure on a complex manifold $X$ is a $J$-invariant 2-form $\Omega$ satisfying $d\Omega = \theta \wedge \Omega$ for some closed 1-form $\theta$. When the Lee field $\xi$ dual to $\theta$ is *holomorphic Killing* with respect to the Hermitian metric $g = \Omega \circ J$, $X$ is said to be a *Vaisman* manifold. We have shown that any simply connected homogeneous Vaisman manifold is the product of $\mathbb{R}$ with a regular Sasaki manifold up to *modification*. This is based on our work on homogeneous locally conformal Kähler manifolds with D. A. Alekseevsky, V. Cortés, K. Hasegawa and myself.

Time：11:00-12:00, Friday, February 17, 2017
Venue：Seminar Room 617, Institute of Mathematics (NTU Campus)
Organizer：Prof. Jih-Hsin Cheng (Academia Sinica)
Refreshment：10:30

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