National Center for Theoretical Sciences

Time: March 19 - June 11, 2024
Every Tuesday, 15:30-17:10

Venue: R440, Astronomy-Mathematics Building, NTU

Adjoint L-values and Tate Conjectures

Speaker: Haruzo Hida — NCTS Distinguished Scholar; NTU Math. Visiting Chair; University of California, Los Angeles

Organizer: Ming-Lun Hsieh — National Taiwan University

Registration

Introduction

There are many fascinating exact formulas in number theory, relating analytic L-values to arithmetic invariants; for example,

(1) Dirichlet's class number formula (proven in 1839 though known to Gauss earlier). Take a complete representative set S of ideal classes of an imaginary quadratic field K. The size $|S|$ is called the class number h_K of an imaginary quadratic field $K = \mathbb{Q}(\sqrt{-d})$ for the discriminant $d > 0$. For each $\mathfrak{a} \in S$, the order $R_\mathfrak{a} := \{a \in K \mid a\mathfrak{a} \subset \mathfrak{a}\}$ is independent of $\mathfrak{a}$ and is the integer ring R of K. The formula is:

$$\frac{\sqrt{-d}L(1,\tau_K)}{2\pi\sqrt{-1}} = \frac{h_K}{R^\times} = \sum_{\mathfrak{a}\in S} \frac{1}{|R_\mathfrak{a}^\times|},$$

where $L(1,\tau_K)$ is the Dirichlet L function for the quadratic character τ_K associated with $K/\mathbb{Q}$.

(2) Siegel's mass formula (proven in 1935). Take a complete representative set S of right ideal classes of a definite quaternion algebra D over Q. The left order $R_\mathfrak{a} := \{a \in D \mid a\mathfrak{a} \subset \mathfrak{a}\}$ depends on $\mathfrak{a} \in S$. The formula is:

$$\mathfrak{m}_1 \frac{\zeta(2)}{\pi^2} = \sum_{\mathfrak{a}\in S} \frac{1}{|R_\mathfrak{a}^\times|}$$

with an explicit rational number $\mathfrak{m}_1$. Here the left-hand-side is called the mass for D.

The purpose of this lecture series is to explain a generalization of the above formulas to the modular adjoint L-values in the simplest case of elliptic curves $E/\mathbb{Q}$.

Course Content

(1) L-value as the size of congruence modules.
(2) Twisted adjoint L-value formula by quadratic characters.
(3) Automorphic forms on quaternions over $\mathbb{Q}$.
(4) Tamagawa number, Siegel-Weil formula and the theta correspondence.
(5) Elliptic adjoint L-value formula.
(6) The Tate conjectures on period and algebraic cycles.

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